MEASURING

OF

GLAZING,	JOINERY,
PAINTING,	CARPENTERS,
PLAISTERING,	and
MASONRY,	BRICKLAYERS

WORKS,

BY

VULGAR ARITHMETICK,

WITHOUT

Reducing the *Integers* into the leaſt Denomination.

The FOURTH BOOK.

By *VEN. MANDEY.*

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MEASURING

OF

CARPENTERS WORK.

CHAP. I.

MENSURATION,

OR Meaſuring, is a Science, whereby we are certified (in ſuperficial Meaſure) either how many Inches, or how many Feet and Inches, or how many Yards, Feet and Inches, or how many Squares, Feet, and Inches, there is in any Dimenſion, whether it be of Glaſs, Board, Stone-paving, Plaiſtering, Painting, Tyling, Carpenters or Joiners Work, *&c.* And ſo conſequently it certifies us of the Content of many Dimenſions, by adding the Products of the ſeveral Dimenſions together.

It doth the ſame in ſolid Meaſure; there is only this Difference between ſuperficial and ſolid Menſuration; to wit, in ſuperficial Meaſure there is only two Sums in a Dimenſion, *viz.* Length and Breadth, to be multiplied one by the other: But in ſolid Meaſure there are three Sums in a Dimenſion; to wit, Length, Breadth, and Thickneſs, to be multiplied each by the other; and this kind of ſolid Menſuration ſerveth for the meaſuring of Timber, Stone, Digging, Bricklayers Work, for it is commonly reduced to a Thickneſs, and all manner of ſolid Bodies whatſoever.

The Inſtruments that are uſed in taking of the Lengths and Heights (or Breadths) in meaſuring of the following Works, are a ten Foot Rod, and a five Foot Rod, and a two Foot Ruler, and ſometimes a Line.

I ſhall begin with the Meaſuring of Carpenters Work, it being moſt eaſy to learn, and ſo proceed through the other Trades, leaving the Meaſuring of Bricklayers Work until the laſt, it being moſt difficult.

CHAP. II.

THE Flooring, Roofing, Partitions, and Ceiling Joiſts, being Carpenters Work, is generally meaſured by the Square (as it is vulgarly ſaid) or more properly is reduced into Squares.

Which Square conſiſts of (or contains) 100 ſuperficial Feet, being the Product of a Square Superficies multiplied in it ſelf, being ten Feet in Length, and ten Feet in Breadth.

Note, That a Superficies is that which hath only Length and Breadth, as a Square incloſed with four Lines on Paper, is called a Superficies (as appears more at large in the precedent Treatiſes of Geometry.)

And from thence the meaſuring of Flooring, Roofing, Partitioning, Ceiling-Joiſting, Boarding, Plaiſtering, Painting, &c. is called ſuperficial Meaſure, becauſe their Thickneſſes are not conſidered, nor taken Notice of in Meaſurement.

For generally Boards, Glaſs, Painting, Plaiſtering, are ſeverally for the moſt part of one Thickneſs.

And although ſome Floors, and Roofs, and Partitions, do require to be made ſtronger than other ſome, by reaſon of the Greatneſs of the Building, and by this means the Timbers are larger and thicker; yet there is no reſpect had to (nor cognizance taken of) the Largeneſs and Thickneſs of the Timbers, in the Meaſurement; but an Allowance is made in the Price, by adding ſo much *per* Square more than is uſually given for ordinary Work.

Thus 12 Inches in Length, and 12 in Breadth, is called a ſuperficial Foot, of Roofing, Flooring, Boarding, Partitioning, Glazing, Painting, Plaiſtering, &c.

Note, When a Carpenter's Bill of Meaſurement is made, there is ſet down,

For ſo many Squares of Roofing (at what Price they agree upon *per* Square) ſo much Money.

Likewiſe for ſo many Squares of Flooring and Boarding, at ſo much *per* Square, ſo much Money.

Alſo for ſo many Squares of Partitioning, at ſo much *per* Square, ſo much Money.

And for ſo many Squares of Ceiling-Joiſts, &c.

The Windows they ſet down either at ſo much *per* Light, or ſo much *per* Window.

The Door-caſes at ſo much a-piece, either with or without Doors.

The Mantletrees and Tarſels at ſo much a-piece.

The Lintolling, Guttering, Cornice, and Window Boards, at ſo much *per* Foot.

Stairs at ſo much a Pair, or ſo much *per* Step, *&c.*

CHAP. III.

IT is ſuppoſed that he that intends to learn the Science of Meaſuring is an Arithmetician.

For although ſeveral Propoſitions may be anſwer'd, ſome Geometrically, and ſome on the Line of Numbers (otherwiſe called *Gunter's* Line) and ſome on the Lines of Superficies and Solids on a Sector; yet without Arithmetick it is impoſſible to meaſure exactly all kinds of Plains and Bodies.

Therefore to the Arithmetician, I ſay, multiply the Length of any Dimenſion (that is, either Square or Oblong) by the Breadth thereof, and the Product is the ſuperficial Area or Content.

I ſhall begin with a Dimenſion of Inches, and ſo proceed gradually to Feet, and from Feet, to Feet and Inches (or Parts.)

Note, That two Sides of a Superficies being meaſured and expreſs'd by Arithmetical Figures (one Sum being the Length, the other the Breadth) is called a Dimenſion.

Example in Inches.

Suppoſe a Piece of Board or Glaſs, or flat Stone, or Painting, or Plaiſtering, or any thing, be it what it will, that is to be meaſured ſuperficially, (vulgarly ly called flat Meaſure) be 11 Inches in Length, and 9 Inches in Breadth (which two Sums 11 and 9, are called a Dimenſion) and you would know the Area or Content thereof;

In.
Set down your Dimenſion thus — 11 the Length.
09 the Breadth.

Then multiply the Length by the Breadth.
And the Product is ———— 99 Inches.

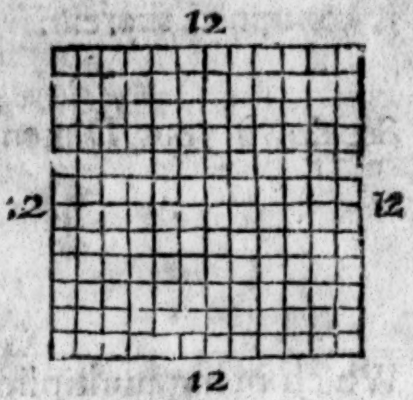
Which 99 Inches is half a Foot and 27 Inches, or very little more than $\frac{2}{3}$ of a Foot.

So that the Product of 11 Inches, multiplied by 9 Inches, which is 99 Inches, contains 8 ſuch Inches, whereof 12 make a ſuperficial Foot, that is to ſay, it contains 8 Inches, whereof each Inch is one Inch in Breadth, and 12 Inches long: For 8 times 12 is 96, which is $\frac{2}{3}$ Parts of a Foot; then the 3 Inches which are remaining, (for the Product was 99 Inches, take 66 from 69, and there remains 3; theſe three Inches, I ſay) are $\frac{3}{2}$, or, as I call them, 3 Parts of ſuch an Inch as aforeſaid; namely an Inch in Breadth, and 12 in Length.

For, by the way, you muſt note, That in a ſuperficial Foot, there is contained 144 Inches, which is the Product of 12 Inches multiplied by 12, thus ————

12 Length.
12 Breadth.
144 Product.

Or as you may ſee in the Geometrical Figure hereunto annex'd.



By this Geometrical Figure, you may plainly perceive, that there is in a ſquare Superficies, being 12 Inches on each Side, 144 Superficial Inches; for every little Square in the Geometrical Figure repreſents an Inch: So conſequently in half a Foot there is 72 Inches, being produc'd from an Oblong, being 12 Inches in Length, and 6 in Breadth.

Which Oblong you may perceive, or imagine in this ſquare Geometrical Figure, by taking the whole Length one Way, and half the Length, or 6 of the little Squares, the other way: Therefore I ſhall not need here to deſcribe the Figure of an Oblong.

Alſo half a ſuperficial Foot, which is 72 Inches, is produced from an Oblong very near a Square, being 9 Inches in Length, and 8 Inches in Breadth, for 9 times 8 is 72.

Likewiſe a Quarter of a ſuperficial Foot contains 36 Inches, which is one half of 72 Inches, or the Product of 6 Inches, multiplied by 6 Inches, or 9 by 4.

I ſhall now proceed to Dimenſions in Feet.

Suppoſe there is a Timber Floor (or a Stone Pavement, or a Plaiſter'd Ceiling, or a Piece of Wainſcot, or any other Superficies) that is 22 Feet in Length, and 18 Feet in Breadth, and you would know ly called flat Meaſure) be 11 Inches in Length, and 9 Inches in Breadth (which two Sums 11 and 9, are called a Dimenſion) and you would know the Area or Content thereof;

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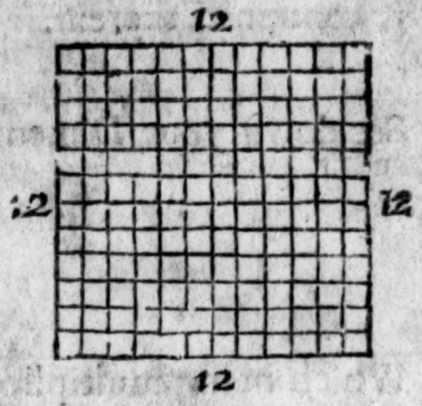

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Feet.
Set down your Dimenſion thus —— —— $\left\{ \begin{array}{r} 22 \\ 18 \end{array} \right.$

176
22

Which being multiplied, the Product is —— 396 ſuperficial Feet.

Now to bring theſe Feet into Squares, you muſt divide 396, the Content in Feet by 100 (the Content of a Square whoſe Side is 10 Feet) and the Quotient is 3 Squares, and 96 Feet remaining.

(96 Feet.
See the Example —— —— $\cancel{3}\cancel{9}\cancel{6}$(3 Squares.
$\cancel{1}\cancel{0}\cancel{0}$

Here you ſee by the Example, that there is 4 Squares of Flooring, wanting 4 Feet; or 3 Squares and three Quarters of a Square, and 21 Feet.

If the Dimenſion had been Stone Pavement of Maſonry, then there had needed no Diviſion, for they work by the Foot; ſo there had been 396 Feet of Pavement with broad Stone, at ſo much *per* Foot.

But if it had been Plaiſtering, or Painting, or Wainſcoting, or Hangings, or ſuch like; then you muſt have brought the Feet into Yards, which is done by dividing 396 Feet by 9 Feet, which is the Number of Feet contained in a ſuperficial Yard (or Quadrate) being 3 Feet in Length, and 3 Feet in Breadth.

See

3
See the Example———————— 396 (44 Yards.
99

Thus having divided 396 Feet by 9 Feet (or one Yard) you have 44 Yards in the Quotient, which is the Number of Yards contained in a Superficies of Plaiſtering, or Painting, or Wainſcot, or Hangings, or ſuch like, being 22 Feet in Length, and 18 Feet in Breadth.

Another Example in Feet.

Suppoſe a Floor or Partition be 146 Feet in Length, and 97 Feet in Breadth.

Feet.
Set down the Dimenſion thus —— 146
97
———
1022
1314
———
Being multiplied, the Product is——14162 Feet.
———

Which being divided by 100, produceth 141 Squares, and 62 Feet: Or you need not divide it, but read it thus, One hundred forty, one hundred and ſixty two Feet, which is 141 Squares and an half, and 12 Feet.

If you were to bring the ſame Dimenſion into Yards, then you muſt divide the Product of the Multiplication, being 14162, by 9 Feet (or one Yard) as you did in the foregoing Example.

563 (5 Feet.
See this Example——— —— 14162 (1573 Yards.
9999

So you have in the Quotient 1573 Yards and 5 Feet remaining, which is half a Yard and half a Foot Thus much may ſerve for meaſuring of Dimenſions of whole Feet.

I ſhall now proceed to Dimenſions in Feet, and Inches, which is ſomething more difficult.

Suppoſe a Floor, or Partition, or any other ſuperficial thing, as Plaiſtering, or Painting, &c. be 21 Feet, and 6 Inches in Length, and 15 Feet, and 6 Inches in Breadth; and it is required to know how many ſuperficial Feet there is contained therein, and conſequently how many Squares, or Yards.

Set down your Dimenſion thus ——

$$\begin{array}{rcl} \text{Feet} & & \text{In.} \\ 21 & \diagdown\!\!\diagup & 06 \\ 15 & \diagup\!\!\diagdown & 06 \\ \hline 105 & & \\ 21 & & \\ \hline \end{array}$$

Then multiply the Feet, and the Product is ——— ——— ——— } 315.

Then for the 6 Inches in Length, and 6 in Breadth, multiply them Diagonal or Croſs-wiſe into the Feet, ſaying 15 times 6 Inches, (or 6 times 15 Inches) is 7 Feet and 6 Inches: Or thus more briefly, (6 Inches being half a Foot) ſay, The half of 15 Feet is 7 Feet and 6 Inches, which you muſt add to the former Product 315. This being done, in the next place you muſt multiply the 6 Inches in the Breadth into the 21 Feet in the Length; ſaying, The half of 21 Feet is 10 Feet and 6 Inches, which you muſt likewiſe add to the reſt.

Then laſt of all, you muſt multiply the Inches of the Breadth into the Inches of the Length, ſaying, 6 times 6 is 36, which 36 is accounted but for 3 Inches, the Reaſon whereof, in the next following Page but one, ſhall be ſhewn.

For Note, How many times 12 you have in the Product of the Inches, being multiplied in themſelves, ſo many Inches you muſt add to the former Work, as in this Example, you have 3 times 12 in 36: As you may ſee in the following Example.

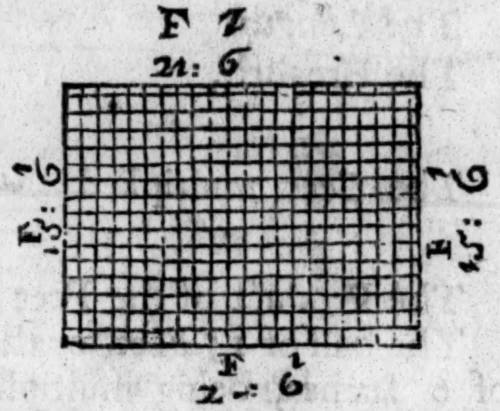
Whence Note, That it is uſual to begin the Multiplication towards the left Hand firſt, namely, the Feet into the Feet, and afterwards the Inches into the Feet, &c.

Example of Operation.

	Feet	In.
The Length,	21	6
The Breadth,	15	6
The Feet multiplied —— ——	105 21	
The Product of the Feet —— ——	315	
The half of 15 Feet, or the Product of 6 Inches, being multiplied by 15 Feet —— —— ——	7	6
The half of 21 Feet, or the Product of 21 Feet, being multiplied by 6 Inches	10	6
The Product of 6 Inches multiplied by 6 Inches, which produces 36, or 3 times 12, which, as you read before, is 3 Inches —— ——		3
	333	3

Then add your Sums together, beginning at the Inches on the right Hand, ſaying, 3 and 6 is 9, and 6 is 15 Inches, which is 1 Foot and 3 Inches; then ſet down 3 under the Column of Inches, and carry 1 to the Feet, and ſay, 1 that I carry and 7 makes 8, and 5 makes 13; then ſet down 3 in the place of Units, under Feet, and carry one to the next place, ſaying, 1 that I carry and 1 is 2, and 1 makes 3, which I ſet down on the left Side of the former 3 (to wit) under the place of Tens: Then I proceed in the next place to 3, which being in the place of Hundreds is 300, and ſet that 3 down on the left Side of the two other.

So the whole Product of 21 Feet and 6 Inches, being multiplied by 15 Feet and 6 Inches, is 333 Feet and 3 Inches, as you have it above in the Example: I think it convenient here to add a Geometrical Figure for the better underſtanding of what hath been ſaid.



By this Geometrical Figure, you may perceive the Truth of what hath been ſaid.

Firſt, Here you ſee that the whole Squares or Feet are 21 in Length, and 15 in Height or Breadth, which produce 315 Feet.

Secondly, In the upper Part of the Figure, you ſee 21 half Squares or half Feet, which is, the 6 Inches which you multiplied by the 21 Feet, the Product whereof was 10 Feet and an Half; ſo in the Figure, accounting two half Squares (or half Feet) for one whole one, you will find 10 Feet and an Half, as before by Arithmetick.

Thirdly, On the right Side of the Figure, you ſee 15 Oblongs or half Feet, which are the ſix Inches that you multiplied by the 15 Feet, which did produce 7 Feet and an half; ſo in the Figure, accounting two halfs for one whole, you will have 7 Feet and an Half, as you had before by Arithmetick.

Fourthly, In the uppermoſt Angle, on the right Side of the Figure, you ſee a little Square, which is 6 Inches in Length, and 6 Inches in Breadth: This is the 6 Inches that you multiplied by 6 Inches, which produced 36 Inches, for which there was ſet down but 3 Inches: The Reaſon whereof is this;

Theſe 3 Inches are the ¼ of 12 Inches, ſo is 36 the ¼ of 144 Inches, of which I have declared before, that there is 144 ſquare ſuperficial Inches in a ſuperficial Foot.

Therefore for Conveniency and Brevity in adding ſeveral Dimenſions together, the odd Inches being multiplied in themſelves, 144 Inches are accounted as 12 Inches and 12 of 144 are accounted as one Inch.

Now if you will give your ſelf the Trouble to tell the Squares in the Geometrical Figure, which Squares repreſent Feet, and the half Squares or Oblongs which repreſent half Feet, and add them together, you will find that they make 333 Feet beſides the little Square in the upper Angle of the Figure on the right Side, which repreſents 3 Inches or ¼ of a Foot; ſo you will have 333 Feet and 3 Inches, as was produced by Arithmetick before.

By what hath been declared, I think it eaſy for a mean Capacity to underſtand how to caſt up the foregoing Dimenſion, and bring it into Feet.

Now to know how many Squares of Carpenters Work there are in this Dimenſion, whoſe Product is 333 Feet and 3 Inches; you muſt either divide 333 by 100, or elſe cut off the two laſt Figures with a Stroke thus 3|33 and read it thus, 3 Squares, and 33 Feet and 3 Inches.

So in 14162 Feet, cut off the two laſt Figures thus; 141|62, and then you read it 141 Squares and 62 Feet.

So in other Numbers, conſiſting of three Figures or more, cut off the two laſt Figures to the right Hand, and the remaining Figures will be Squares.

But if you would know how many Yards there is in 21 Feet and 6 Inches in Length, and 15 Feet and 6 Inches in Breadth; you muſt, as before is taught, divide the Product of the Dimenſion, being 333 Feet, by 9, being the Number of Feet in a ſquare Yard ſuperficial.

As in Example,

$$\begin{array}{ccccl} & \cancel{6} & & & \\ \cancel{3} & \cancel{3} & \cancel{3} & \Big(& 37 \text{ Yards.} \\ & \cancel{9} & \cancel{9} & & \end{array}$$

So your Quotient is 37 Yards; the odd 3 Inches muſt be added after you have divided.

Take another Example.

Suppoſe you have a Floor or Partition, that is 35 Feet and 10 Inches in Breadth, and 46 Feet 9 Inches in Length. Set down your Dimenſion as before, and multiply the whole Feet 46, by the whole Feet 35; and the Work will ſtand thus,

$$\begin{array}{rcr} \text{Feet.} & & \text{In.} \\ 46 & \times & 09 \\ 35 & & 10 \\ \hline 230 & & \\ 138 & & \end{array}$$

Secondly, Multiply the Inches into the Feet croſs-ways, ſaying 9 times 35 is 315, which muſt be ſet apart by it ſelf, as you may ſee in the Example in this Page.

Thirdly, You muſt multiply the 10 Inches by the 46 Feet, which produces 460; this you muſt add to the 315, which before you ſet apart, and they make 775 Inches, which you muſt divide by 12, to bring the Inches into Feet, which being divided, the Quotient is 64 Feet, and 7 Inches remaining, which 64 you muſt add to the Multiplication of the Feet, and the 7 to the Inches.

See the Example of the Inches multiplied into the Feet.

46 09
35 10

The Product of the 10 Inches being multiplied into 46 Feet is 460 Inches

The Product of 9 Inches multiplied by 35 Feet is 315 Inches

Which being added, is 775 Inches

The 775 Inches being divided by 12, produces, as in the Margin, 64 Feet and 7 Inches, which muſt be added to the Multiplication of the Feet in the preceding Page, and then the Work will ſtand thus:

~~1~~
~~1~~ ~~5~~(7
~~7~~ ~~7~~ ~~5~~(64
~~1~~ ~~2~~ ~~2~~
~~1~~

46 09
35 10

230
138
64 7
7

Fourthly, You muſt multiply the 10 Inches belonging to the Breadth, by the 9 Inches belonging to the Length, and the Product will be 90 Inches of 144 Inches; or 7 Inches and an Half of 12 Inches, for if you divide 90 by 12, you will have 7 in the Quotient, and 6 remaining, which 7 in the Quotient is 7 Inches, and muſt be added to the Place of Inches; and the 6 that remains over and above the Quotient, being the ½ of 12, is half an Inch, which is ſeldom accounted in Carpenters Work: But although it is not uſually done, yet if you are minded to be exact in Meaſuring, you may ſet down theſe odd Parts of Inches by themſelves, and at laſt add them all together, and for every 12 of theſe Parts you may add 1 Inch, and for 144 of them you muſt add 1 Foot, as you read before.

Then adding all your Sums together, you find that 46 Feet 9 Inches, multiplied by 35 Feet 10 Inches, is 1675 Feet 2 Inches.

Which if you are to bring into Squares, you cut off the two Figures next the right Hand thus 16|75 and then you read it 16 Squares and 75 Feet (or 16 Squares and ¾ of a Square.)

An Example of the whole Work.

	Feet	In.
The Length,	46	09
The Breadth,	35	10
The Product of 46 by 5,	230	
The Product of 46 by 3,	138	
The Product of all the Inches multiplied by all the Feet,	64	07
The Product of the 10 Inches multiplied by 9,		07
The whole Product	16|75	02

Alſo by the way, take Notice that Girders Ends, and Ends of Breſt-Sommers, and Plates, and ſuch like, muſt be remember'd and ſet down in your Bill of Meaſurement.

Alſo you muſt allow in your Meaſure, for the Ends of the Joiſts that lie in the Walls; if it be Flooring unboarded that you are to meaſure, then you ought to allow 9 Inches into each Wall that way the Joiſt Ends are laid.

But if you meaſure Flooring and Boarding, then if you allow 6 Inches, it is reaſonable, becauſe though the Timbers go into the Walls 9 Inches, yet the Boarding goeth but home to the Wall.

And as you add theſe Things, you muſt alſo remember to deduct the Stairs and Chimneys where the Workman findeth Materials, elſe not.

Thus much will ſerve as to the Meaſuring of Flooring, Partitioning, and ſuch like; In the next Place I ſhall treat

Of the Meaſuring of Roofs.

Suppoſe a Building to be 30 Feet in Breadth, from the Outſide of one Wall to the Outſide of the other, and 65 Feet in Length, from out to out of the Walls, and you would know how many Squares of Roofing there are in the Roof of this Building.

If the Roof be true Pitch, you need not meaſure the Length of the Rafter, but take this for a general Rule:

That the Length of the Rafter is $\frac{3}{4}$ of the Breadth of the Building, therefore the Breadth being 30 Feet, the Length of the Rafter will be 22 Feet and 6 Inches, which is $\frac{3}{4}$ of 30) which being doubled (for ſo it muſt be for both Sides) makes 45 Feet: Then multiply 45 by the Length of the Building being 65, and the Product will be 29|25, which is 29 Squares and a Quarter.

But the uſual way, which is ſomething briefer, is thus: Multiply the Length 65 by the Breadth 30, and that produces 1950 Feet for the flat of the Building; then add half the Number of Feet in the flat (to wit) 975 to 1950, and the Product is 2925 as before.

You may add the half thus, ſaying, The half of 19 is 9, then becauſe twice 9 is but 18, and there remains 1; you muſt carry that one to the next Figure being 5, and for that 1, which remained of the 19 after it was halfed, you muſt add 10 to 5, which makes it 15.

Then ſay, The half of 15 is 7 and 1 remaining, add that 1 to the Cypher, and it makes 10.

Then ſay, The half of 10 is 5.

See both the Examples.

	Feet	In.
The Length of the Rafter on one Side of the Roof	22	6
The Length of the Rafter on the other Side	22	6
The Length of both Rafters being added	45	0
The Length of the Building to be multiplied by 45	65	0
	225	
	270	
The Product	2925	0

An

An Example of the other way, called Menſuration on the Flat.

	Feet	In.
The Length,	65	0
The Breadth,	30	0
The Product of the Length by the Breadth on the Flat,	1950	0
The half Flat to be added to the Flat,	975	0
The Product of the Flat and half	2925	0

Note, When you meaſure Roofing this laſt way, being called Meaſuring on the Flat, you muſt write over your Dimenſions, *Flat and Half*; or 1 ½ to ſignify, that after you have multiplied the Length by the Breadth, you muſt add half that Product to it ſelf.

Note, If the Building that you are to meaſure be broader at one End than it is at the other, you muſt meaſure the Breadth in the Middle, both for the Flooring and the Roofing; or elſe meaſure each End, and add the Sums together, and take half the Product: As, ſuppoſe a Building to be 14 Feet wide at one End, and 12 Feet wide at the other, they being added, make 26, the half whereof is 13 Feet for the Width of the Building.

How to meaſure a Gable-end.

Suppoſe a Timber Building be 40 Feet in Breadth, and you have meaſured all the Carcaſs of the Building, except the Gable-end, or Ends;

To meaſure this Gable-end, multiply half the Baſe by the whole Perpendicular; or half the Perpendicular by the whole Baſe, gives you the Area, or Content of it.

I call the Plate, whereon the Rafters ſtand, the Baſe.

The Baſe being 40 Feet, the Perpendicular is 22 Feet and 4 Inches.

See the Example.

	Feet	In.
The half of the Baſe,	20	0
The whole Perpendicular,	22	4
The Product of 2 multiplied by 20,	40	0
The Product of 20 by 20,	400	0
The Product of the 4 Inches multiplied by the 20 Feet,	6	8
The total Product,	446	8

The other Example.

	Feet.	In.
The whole Baſe,	40	0
The half of the Perpendicular,	11	2
The Product of 40 by 1,	40	0
The Product of 40 by 10,	400	0
The Product of the 2 In. by the 40 Feet,	6	8
The total Product,	446	8

Thus you ſee both ways agree, and the Gable-end contains 4 Squares and 46 Feet 8 Inches. Iſ

If there be two Gable-ends equal, or both alike in the Building, then you muſt ſet down the total Product twice.

If there be four Gable-ends, and each of them of one Bigneſs, then you muſt ſet down the total Product four times.

Note, If the Building be ſquare (*viz.*) hath four right Angles, and the Breadth 40 Feet; and the Roof true Pitch;

The Length of the Rafter, and alſo of the Hip Rafter, and the Angles which they make; alſo the Length of the Diagonal Line, and of the Perpendicular, are as in the following Table.

As you may ſee by the Figure A B C D hereunto annexed.

	Feet	In.
The Breadth of the Building,	40	00
The Length of the Rafter,	30	00
The Length of the Hip Rafter,	36	00
The Length of the Diagonal Line,	56	06 $\frac{2}{3}$
The Length of the Perpendicular,	22	04 $\frac{1}{4}$

		Deg.	Min.
Rafter Angles	At Foot	48	10
	At Top	41	50
Hip Angles	At Foot	38	22
	At Top	51	38

Explanation of the Fig. ABCD.

A B. The Breadth of the Building.
A P. The half Breadth.
A E, or E B, The Length of the Rafter.
E P The Perpendicular.

A F, or F B, The Length of the Hip Rafter.

A G H, The Angle that the Hip makes at Top, and K *i*, the Meaſure of it.

H A G, The Angle that the Hip makes at the Foot, and L I the Meaſure of it.

P A E, The Angle that the Rafter makes at Foot, and P *a* the Meaſure of it.

A E P, The Angle that the Rafter makes at Top, and *a*, *n*, the Meaſure of it.

L I P, The Quadrant of a Circle divided into 90 Degrees, for the meaſuring of the Angles.

A C, The Diagonal.

L M, The Line over which the Rafters muſt be placed, that the upper Ends of the Hip Rafters are fixed to.

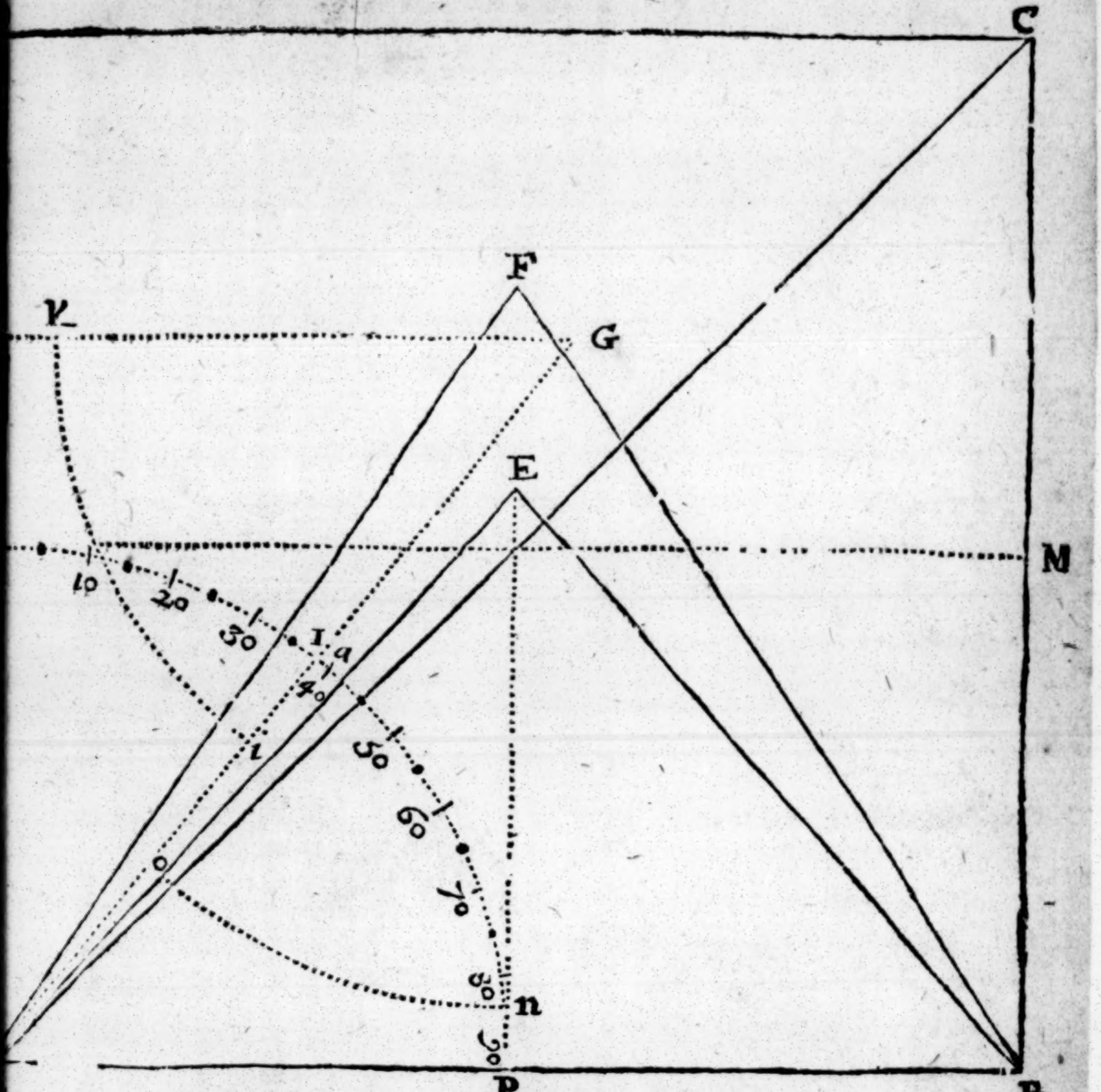
The Length of the Hip Rafter is found, by taking the Length of the Rafter A E, and ſetting it on the Perpendicular from P to F, draw F A the Hip Rafter.

If you have a Roof of any other Width, you may deſcribe it from this Figure, or you may find the Length of the Hip Rafters, and the Length of the Diagonal, and likewiſe of the Perpendicular by the Rule of Three, and the foregoing Table.

The Angles are always equal to thoſe in the Table, let the Width be what it will, provided the Building be right angled, and the Roof the uſual true Pitch (that is, the Rafters Length to be $\frac{3}{4}$ of the Breadth of the Building.

To Meaſure a Hipt Roof.

Suppoſe a Building to be 60 Feet in Length, and 40 Feet in Breadth, ſtanding alone, being Hipt from each



Place this between Page 208 and 209.

each Angle, the Angles being right, and the Roof true Pitch, and it is required to know how many Squares of Roofing there is in it.

Multiply the whole Length, being 60 Feet, by 30 Feet, the Length of the Rafter, for one Side of the Roof.

Or add 30 and 30 together, they make 60 for both Sides of the Roof, then multiply 60 by 60, the Product is 3600, or 36 Squares in that Hipt Roof.

Which I thus demonſtrate.

The Length of the Roof on the Ridge, from Hip to Hip is 20 Feet, and the Length of the Roof at the Plate is 60 Feet.

Add theſe two Sums together, and they make 80 Feet, whereof take half, that is 40 Feet, which is the mean Length of the Roof, if you meaſure in the Middle between the Top and Bottom; then multiply this 40 Feet by 60 Feet, being the Length of both Rafters on each Side the Roof, and the Product is 2400, which is the Meaſure of the two Sides of the Roof.

Proceed we, in the next Place, to the two Ends;

Being each of them 40 Feet in Length, and the Rafter of each End 30 Feet in Length; each of the Ends being a Triangle, which you muſt meaſure, as you did the Gable-end.

Multiply 40 Feet being the Baſe, by 15 Feet being one half of the Rafter (or Perpendicular as you may call it, although it lean from an upright) and the Product is 600.

The other End of the Roof being the ſame as this, add 600 to 600 it makes 1200, which being added to 2400, the Product of the two Sides, make 3600, or 36 Squares as before, which was to be demonſtrated.

Suppoſe you were to meaſure the Roofing of a corner Houſe, being 60 Feet long, and 40 Feet broad, and joining to other Houſes each Way, then there will be but one Hip, and one Sleeper, yet the Dimenſions will be the ſame, and the ſame Number of Squares in it, as in the former that was Hipt, provided the Building be Square, and the Roof true pitch.

See the Examples.

	Feet	In.
The whole Length of the Roof,	60	00
The Length of both Rafters,	60	00
The Product or Number of the Squares	3600	00

The Demonſtration.

	Feet	In.
The Length of the Roof on the Top is	20	00
To which add the Length of the Roof at the Plate	60	00
The Sum is	80	00
The Length of both Rafters	60	00
Being multiplied by 40, the Mean between 60 and 20	40	00
The Product is Which is the Content of the two Sides.	2400	00

Now for the two Ends.

	Feet	In.
The Length of the Baſe,	40	00
Multiplied by the half Length of the Rafter,	15	00
	200	00
	40	
Produces	600	00
To which add the Meaſurement of the other End,	600	00
The Sum is the Content of both the Ends,	1200	00
To which add the Meaſure of both Sides,	2400	00
The Sum is the Area or Content of the whole Roof, as before,	3600	00

How to find the Length of the Hip for a Roof that is 20 Feet in Breadth, by the Rule of Three.

You ſee by the foregoing Figure A B C D, that a Roof being 40 Feet in Width or Breadth, gives a Hip Rafter that is 36 Feet in Length; therefore ſtate your Queſtion thus: If 40 give 36, what will 20 give?

Multiply and Divide, and you will find 18

$$\begin{array}{ll} 36 & \cancel{3} \\ 20 & \cancel{7}\cancel{2}\cancel{0}\,(18 \\ \hline 720 & \cancel{4}\cancel{0}\cancel{0} \\ & \cancel{4} \end{array}$$

After the ſame Manner you may find any of the reſt. You muſt remember to add the Rafters Feet, and Eves Boards, in the Bill of Meaſurement.

CHAP.

CHAP. IV.

Meaſuring of Glaziers Work.

THE Meaſuring of Glaziers Work is the ſame as of Carpenters Work; only Carpenters Work is not meaſured any nearer than two whole Inches.

But Glaziers Work is ſometimes meaſured to $\frac{1}{4}$ of an Inch; and the beſt and readieſt way to meaſure Glazing, is to take the Dimenſions with a ſliding Ruler, ſuch as Glaziers generally uſe, which Ruler is divided Decimally, a Foot into 100 Parts, ſo that a whole Foot contains 100 Parts or Diviſions; $\frac{3}{4}$ of a Foot contains 75 Parts; $\frac{1}{2}$ of a Foot contains 50 Parts,, and $\frac{1}{4}$ of a Foot contains 25 Parts.

Thoſe that deſire to meaſure the Decimal Way, I refer them to a Treatiſe put forth by my Friend *Tho. Hammond* (Entituled, *A new and exact Way of Menſuration*) which I writ for him when I was Apprentice, and indeed it is a very ready Way of Meaſuring, provided our two Foot Rules were divided into 20 equal Parts, or a Foot Rule into 10 Parts (or, as I may call them, Inches) and every one of thoſe Parts divided into 10 equal Parts.

But to come to the Thing intended (to wit) to meaſure Glazing by vulgar Arithmetick, and with a two Foot Ruler divided into 24 Parts, and each of thoſe into 8 Parts, as they are commonly made and uſed.

Suppoſe you have a Pane of Glaſs which is 5 Feet 8 Inches and an half in Length, and 5 Feet 7 Inches and $\frac{1}{4}$ in Breadth, and you would know the Area or Content thereof:

Set down your Dimenſions thus,

Feet	In.	Parts.
5	8	6
5	7	3

And read it, 5 Feet 8 Inches and 6 Parts of 12, which is the $\frac{1}{2}$ Inch; by 5 Feet 7 Inches, and 3 Parts of 12, which is the $\frac{1}{4}$ Inch.

Then multiply the Feet and Inches one by the other, as you are taught in the foregoing Pages, ſaying, 5 times 5 is 25 Feet; which being ſet down under the Feet, multiply croſs-ways 5 by 8, which is 40 Inches, or 3 Feet and 4 Inches. This being ſet down,

In the next Place, multiply 7 by 5, which is 35 Inches, or 2 Feet and 11 Inches. This being ſet down,

In the next place multiply the 7 Inches by the 8, it is 56 Inches, of which 144 make a Superficial Foot, for which you muſt ſet down 4 Inches, becauſe you can have but 4 times 12 out of 56, and there remains 8. Therefore I ſet down 4 Inches and 8 Parts, of which Parts 12 make an Inch, 12 of which Inches are accounted a ſuperficial Foot.

In the next place multiply the Parts of Inches croſs-ways; firſt into the Feet, ſaying, 6 times 5 make 30, which is 2 Inches and 6 Parts. This being ſet down,

Say 5 times 3 is 15, or 1 Inch and 3 Parts, which you muſt put alſo under the reſt.

Then multiply the Parts of Inches croſs-ways into the Inches, ſaying, 6 times 7 is 42, which is 3 Parts and an half, for which I ſet down 3 under the Place of Parts of Inches, and 6 againſt it towards the right Hand, which ſignifies $\frac{1}{2}$ Part of a Part (for 12 of theſe Parts make one ſuch Part, whereof there are

12 in an Inch) Line Meaſure. This being done, multiply 3 by 8 (which is 3 Parts by 8 Inches) that makes 24 Parts, for which you muſt ſet down 2 under the Place of Parts of Inches.

Note, That as in multiplying the Inches by the Inches, you account every 12 of the Product for 1 Inch, ſo in multiplying the Parts of Inches into the Inches, you muſt account every 12 of the Product for 1 Part; as in multiplying 3 Parts by 8 Inches produceth 24, which is twice 12, therefore I ſet down 2 under the place of Parts.

Laſtly, you muſt multiply the Parts of Inches by the Parts (if you will meaſure ſo near) ſaying, 3 times 6 is 18, which is 1 Part and a half of a twelfth Part of an Inch Line Meaſure; ſee the Dimenſion caſt up.

Example.

	Feet	In.	Parts	
	05	8	6	
	05	7	3	
The Product of 5 Feet by 5 Feet is	25	0	0	
The Product of 8 Inches by 5 Feet is	03	4	0	
The Product of 7 Inches by 5 Feet is	02	11	0	
The Product of 7 Inches by 8 In. is	00	4	8	
The Product of 6 Parts by 5 Feet is	00	2	6	
The Product of 3 Parts by 5 Feet is	00	1	3	Pa.
The Product of 6 Parts by 7 Inch. is	00	0	3	6
The Product of 3 Parts by 8 Inch. is	00	0	2	0
The Product of 3 Parts by 6 Parts is	00	0	0	1½
The total Product is	31	11	10	7½

Thus you ſee that the Area or Content of 5 Feet 8 Inches and an Half (or 6 Parts of 12) in Length, being multiplied by 5 Feet 7 Inches and $\frac{1}{4}$ (or 3 Parts of 12) in Breadth, is 31 Feet, 11 Inches and 10 Parts of an Inch, accounted to have 12 Parts, and 7 and $\frac{1}{2}$ of one of thoſe Parts; but as for the laſt 7 Parts and $\frac{1}{2}$, they are not worth the ſetting down nor taking notice of, the Value of thoſe Parts being ſo ſmall.

The Truth of this Area or Content may be proved three ſeveral Ways.

Firſt Decimally, Second Geometrically, Thirdly by Vulgar Arithmetick.

Firſt Decimally thus; the Decimal of 8 Inches and $\frac{1}{2}$ is 71, and the Decimal of 7 Inches and $\frac{1}{4}$ is 60 and ſome ſmall matter more; Therefore in Decimals, I ſet down the Dimenſion thus;

$$\begin{array}{r} \text{F.P.} \\ \left\{ \begin{array}{r} 571 \\ 560 \end{array} \right. \\ \hline 34260 \\ 2855 \\ \hline 31|97|60 \\ \hline \end{array}$$

And having multiplied it, I find the Product to be 319760, then cut off the 2 Figures next to the Left Hand with a Stroak, as you ſee in the Example, and they repreſent 31 Feet.

Then the Remainder is 2760 of 10000, and by cutting of 2 Figures more thus 97|60 it is 97 of 100 and ſomething more, it is near 98 Parts of 100, 92 of which Parts, is the Decimal of 11 Inches, and the 6 Parts which remain are $\frac{6}{10}$ of an Inch; ſo that by this you ſee the Decimal Way of Meaſuring and this Vulgar doth agree.

Secondly, you may prove it Geometrically, if you deſcribe a Geometrical Figure (after the ſame Manner that the Figure is deſcribed in P. 189 of *Carpenters Work*, being drawn according to a Scale;) 5 Feet

8 Inches and ½ in Length, and 5 Feet 7 Inches and ¼ in Breadth; and then divide the Feet and Inches, and Parts; and you will find it to agree with the Area or Content before produced.

Thirdly, it may alſo be proved by Vulgar Arithmetick thus.

Reduce the Feet and Inches into the leaſt Denomination (to wit) Quarters of Inches, and then multiply them one by another.

To do this, multiply 68 Inches (which are the Inches in 5 Feet and 8 Inches) by 4, and that brings them into Quarters of Inches Line Meaſure, then add 2 to the Product, for the half Inch in the Length.

$$\begin{array}{r} 68 \\ 4 \\ \hline 272 \\ 2 \\ \hline \end{array}$$

Thus in the Length you have 274 Quarters of Inches.

The Breadth being 5 Feet 7 Inches and ¼, multiply 67 (the Inches in 5 Feet and 7 Inches Line Meaſure) by 4, being the Quarters in an Inch Line Meaſure, and to the Product add 1, which is the ¼ Inch in the Breadth.

$$\begin{array}{r} 67 \\ 4 \\ \hline 268 \\ 1 \\ \hline \end{array}$$

And in the Breadth is contained 269 Quarters of Inch.

Thus having brought the Length and Breadth into the leaſt Denomination (to wit, Quarters of Inches) I ſet down my Dimenſion thus, and multiply it:

274
269

2466
1644
548

The Product is 73706 Quarters.

Of which Quarters there are 16 in an Inch ſuperficial Meaſure (to wit, 4 in Length, and 4 in Breadth) and of which Inches there are 144 in a Foot ſuperficial Meaſure (to wit, 12 in Length, and 12 in Breadth.) Therefore I multiply 144, the Inches in a ſuperficial Foot, by 16, the Quarters in a ſuperficial Inch, and

144
16

864
144

In a ſuperficial Foot there is 2304. Quarters.

Then divide 73706, the ſuperficial Quarters of Inches in the whole Dimenſion, by 2304, the ſuperficial Quarters of Inches in a Foot.

$$\begin{array}{ccccc}
 & 2 & 2 & 8 & \\
\cancel{0} & \cancel{4} & \cancel{5} & \cancel{8} & 2 \\
\cancel{7} & \cancel{3} & \cancel{7} & \cancel{0} & \cancel{6} \\
\cancel{2} & \cancel{3} & \cancel{0} & \cancel{4} & \cancel{4} \\
 & \cancel{2} & \cancel{3} & \cancel{0} &
\end{array}\Big(\text{31 Feet.}$$

And the Quotient is 31 Feet, and the Remainder is 2282 Quarters, which are not a Foot.

Therefore divide 2282 by 192 (which is the Number of Quarters contained in 12 Inches in Length, and 1 Inch in Breadth, which 192 is the Product of 12 by 16.)

$$\begin{array}{r}
12 \\
16 \\
\hline
72 \\
12 \\
\text{192 Quar.}
\end{array}
\qquad\qquad
\begin{array}{cccc}
 & 1 & 7 & \\
 & \cancel{3} & \cancel{6} & 0 \\
\cancel{2} & \cancel{2} & \cancel{8} & \cancel{2} \\
\cancel{1} & \cancel{9} & \cancel{2} & \cancel{2} \\
 & \cancel{1} & \cancel{9} &
\end{array}\Big(\text{11 Inches.}$$

And the Quotient is 11 Inches, and 170 the Remainder, which is 170 Parts, or Quarters of an Inch, in Breadth, and 12 Inches in Length; which Inch, as I told you before, contains 192 Quarters.

Wherefore divide this 170 by 16 (to wit, the Quarters that are contained in an Inch in Length, and an Inch in Breadth.)

$$\begin{array}{ccc}
 & 1 & \\
\cancel{1} & \cancel{7} & 0 \\
\cancel{1} & \cancel{6} & \cancel{6} \\
 & \cancel{1} &
\end{array}\Big(\text{10 Parts.}$$

And the Quotient is 10, and the Remainder 10.

Thus you ſee the Product of 5 Feet, 8 Inches, and $\frac{1}{2}$ in Length, being multiplied by 5 Feet, 7 Inches, and ¼ in Breadth, is 31 Feet, 11 Inches, 10 Parts, and $\frac{10}{16}$ of a Part, which $\frac{10}{16}$ of a Part, is in Value the ſame that 7 and ½ is, of a Part containing 12, as you found it by the firſt Way of working in Vulgar Arithmetick.

Another Example.

Suppoſe a Pane of Glaſs be 5 Feet, 3 Inches and ½ long, and 2 Feet, 4 Inches and ½ broad, and you deſire to know the Content thereof.

	Feet.	In.	Parts.
Set down your Dimenſion thus	5	3	6
	2	4	6

Then multiply the Feet and Inches one into another, ſaying, 5 times 2 is 10 Feet; ſet that down under the Feet; then multiply croſswiſe, ſaying, 3 times 2 is 6 Inches; ſet that under the Inches.

Then ſay, 4 times 5 is 20 Inches, or 1 Foot and 8 Inches; ſet the 1 under the Feet, and the 8 under the Inches.

Then ſay, 4 times 3 is 12, which is 1 Inch.

Next multiply the Parts croſsways into the Feet, ſaying, 6 times 2 is 12, which is 1 Inch, which muſt be ſet under the Place of Inches.

Then multiply the 6 Parts by 5 Feet, it makes 30 Parts, which is 2 Inches, and 6 Parts; which being ſet down,

Multiply the Parts into the Inches, ſaying, 6 times 3 is 18, which is 1 Part and an half; for which I put 1 under the Place of Parts, and 6 againſt it towards the Right Hand, which ſignifies ½ or $\frac{6}{12}$ of a Part.

Then multiply the 6 Parts by the 4 Inches, it makes 24, which is two Parts; therefore add 2 under the Place of Parts.

Laſtly, multiply the Parts by the Parts, ſaying, 6 times 6 is 36, which is $\frac{3}{12}$ of a Part; ſet this 3 under the 6 on the Right Hand.

Then adding them all together, the Content is 12 Feet, 6 Inches, 9 Parts, and $\frac{9}{12}$ of a Part; which $\frac{9}{12}$ you need not ſet down as you read before, they ſignifying very little; then it will be 12 Feet, 6 Inches, and 9 Parts, or $\frac{3}{4}$ of an Inch.

See the whole Work.

	Feet.	In.	Parts.	
The Length	05	3	6	
The Breadth	02	4	6	
The Product of 5 Feet by 2 Feet is	10	0	0	
The Product of 3 Inches by 2 Feet is	00	6	0	
The Product of 4 Inches by 5 Feet is	01	8	0	
The Product of 4 Inches by 3 Inches is	00	1	0	
The Product of 6 Parts by 2 Feet is	00	1	0	
The Product of 6 Parts by 5 Feet is	00	2	6	Pa.
The Product of 6 Parts by 3 Inches is	00	0	1	6
The Product of 6 Parts by 4 Inches is	00	0	2	
The Product of 6 Parts by 6 Parts is	00	0	0	3
The Total is	12	6	9	9

Note, That in meaſuring of Glazing, many times in a Building there are ſeveral Window-frames of one bigneſs, and in one Window-frame there are ſeveral Panes of Glaſs of one Bigneſs or Dimenſion; as in a ſix Light Window there are 6 Panes of Glaſs, 3 of them of one Dimenſion, and 3 of another; ſo in a four Light Window there are 4 Panes of Glaſs, 2 of one bigneſs, and 2 of another.

In a ſix Light Window you need meaſure but one Pane of a Sort, and ſet down each Dimenſion; and becauſe there are 3 of a Sort of one bigneſs, ſet down 3 againſt your Dimenſion, which ſignifies 3 times that Dimenſion.

If it be a four Light Window, and have 2 Panes of Glaſs of one bigneſs, and two of another, then ſet down 2 againſt your Dimenſion.

Example.

If you were to meaſure a ſix Light Window, and the upper Panes were each of them 2 Feet 6 Inches in height (or length) and 1 Foot 2 Inches in Breadth; and the lower Panes 6 Feet 4 Inches in Length, and 1 Foot 2 Inches in Breadth; you may make but two Dimenſions of theſe 6 Panes of Glaſs.

	Feet.	In.	Parts.	
Setting them down thus	2	6	0	(3)
	1	2	0	

Feet.	In.	Parts.	
6	4	0	(3)
1	2	0	

The 3 in the Circle thus (3) ſignifies 3 times the Dimenſion that it ſtands againſt; therefore when you have caſt up the Dimenſion, the Product thereof

you

you muſt multiply by 3, and ſet that down for the 3 Panes.

Example.

	Feet	In.	Parts.	
	2	6	0	(3)
	1	2	0	
	2	0	0	
		6		
		4		
		1		
The Product of one Pane is	2	11	0	
Which multiply by 3, becauſe there are 3 Panes of that bigneſs		3		
And the Product is	8	9	0	

Being the Content of the 3 Panes.

Then the other 3 Panes, being each of them 6 Feet 4 Inches in Length, and 1 Foot 2 Inches in Breadth,

	Feet	In.	Parts.	
Set down thus	6	4	0	(3)
And multiply	1	2	0	
The Product of 6 Feet by one, is	6	0	0	
The Product of 2 Inches by 6 Feet is	1	0	0	
The Product of 4 Inches by 1 Foot is	0	4	0	
The Product of 2 Inches by 4 Inches is	0	0	8	
The Total Product or Content is	7	4	8	
Which multiply by			3	

Saying

	Feet	In.	Parts.
	7	4	8
			3
Saying 3 times 8 is 24 Parts or 2 Inches, which carry to the Inches, and ſet a Cypher under the Parts, thus			0
Then ſay 3 times 4 Inches is 12 Inches, and 2 that I carry is 14 Inches, that is, 1 Foot, and 2 In. ſet down the 2 In. and carry the 1 Foot to the Feet		2	
Then ſay 3 times 7 is 21 Feet, and 1 that I carry is 22 Feet; ſet that down	22	0	0
Add them, and the Product is	22	2	0

For the Area, or Content of the 3 Panes.

	Feet	In.	Parts.
The Content of the firſt 3 Panes is	08	9	0
The Content of the laſt 3 Panes is	22	2	0
Being added together is	30	11	0

Which is the Content of all the ſix Panes in the ſix-light Window.

Then ſo many ſix-light Windows of the ſame bigneſs as you have in the Building to meaſure, multiply the Number of them by 30 Feet and 11 Inches, and the Product is the Content of them all.

To meaſure a four-light Window.

A four-light Window having 2 Panes of one bigneſs, and 2 Panes of another; you may ſet down but 1 Pane of each bigneſs, and (2) againſt it thus.

Suppoſe the upper Panes to be 2 Feet 1 Inch in length, and 1 Foot 8 Inches and ½ in breadth, a-piece;

And the lower Panes to be 4 Feet in length, and 1 Foot 8 Inches and ½ in breadth a-piece.

Begin with the two upper Panes.	Feet	In.	Parts.	
And ſet them down thus	2	1	0	(2)
	1	8	6	
	2	0	0	
	1	4	0	
		1		
		0	8	
		1	6	
The Content of 1 Pane is	3	7	2	
The other Pane being the ſame, add	3	7	2	
The Content of both the upper Panes is	7	2	4	

Secondly, Set down the Dimenſion of the two lower Panes thus,

	Feet	In.	Parts.	
	4	0	0	(2)
	1	8	6	
	4	0	0	
	2	8	0	
		2		
The Content of one Pane is	6	10	0	
And becauſe there is (2) ſtands againſt the Dimenſion, you muſt multiply it by 2, or elſe add	6	10	0	
The Content of both Panes is	13	8	0	
To which add the Content of the upper Panes	7	2	4	
And the Content of the 4 Panes in the 4 Light Window is	20	10	4	

How to ſet down Dimenſions in your Pocket-Book.

Note, Before you begin to ſet down your Dimenſions, it is convenient to divide the Breadth of your Page or Leaf into ſo many ſeveral Columns as you think convenient, with Lines drawn of Ink: The Leaves of your Pocket-Book being of the Breadth of this Book, you may divide a Leaf into four Parts or Columns.

You muſt likewiſe before you ſet down any Dimenſions, expreſs the Workmaſter, and the Workmens Names, alſo the Place where, and the Day of the Month, and Date of the Year that you meaſure. Likewiſe if the Work that you are to meaſure, be Glazed with ſquare Glaſs, you muſt write *Squares* above your Dimenſions, and over thoſe Dimenſions which are glazed with Quarries, you muſt write *Quarries*; that when you come to make the Bill of Meaſurement, you may expreſs them ſeverally, becauſe they are of ſeveral Prices.

In the next Page I will ſet down all the Dimenſions which you have been taught to caſt up in Glazing, and ſome others, with the Product to each Dimenſion.

Glazing done by A. B. *for* C. D. *in* Long-Acre, *and meaſured the 22d of* January, 1727.

Quarries.	Products.	Squares.	Products.
F. I. P.	F. I. P.	F. I. P.	F. I. P
5 8 6 5 7 3	31 11 10	4 3 0 1 2 0	04 11 06
5 3 6 2 4 6	12 06 09	2 0 0 1 6 0	03 00 00
2 6 0 1 2 0 (3)	08 09 00	3 0 0 2 3 0	06 09 00
6 4 0 1 2 0 (3)	22 02 00	6 0 9 5 0 3	30 05 03
2 1 0 1 8 6 (2)	07 02 04	1 2 0 3 0 0 (2)	07 00 00
4 0 0 1 8 6 (2)	13 08 00		52 01 09
	96 03 11		

Explanation of the Columns.

In the firſt Column towards the Left Hand, are the Dimenſions (which you have been taught to caſt up) of Glazing done with Quarries.

In the ſecond Column you have the Product of each Dimenſion juſt againſt it.

In the third Column you have 5 Dimenſions of Glazing done with Square Glaſs.

In the laſt Column you have the Product of each Dimenſion juſt againſt it; and at the Bottom you have the total Sum of all the Products in that Column, being 52 01 09.

Likewiſe at the Bottom of the ſecond Column you have the total Sum of all the Products of the Dimenſions done with Quarries, which is 96 Feet 3 Inches and 11 Parts. As for the odd Parts, you may leave them out when you make your Bill of Meaſurement.

When you are meaſuring, and ſetting down the Dimenſions in your Book, whether it be of Glazing, or any other Trade, you muſt leave every other Column vacant or empty, that ſo having ſet down all your Dimenſions in your Book (which you generally do before you caſt up any) when you caſt them up (which muſt be in another Book or Sheet of Paper) you may enter the Product of each Dimenſion juſt againſt it, as you ſee in the Page before.

If there be another to meaſure againſt you, you muſt, after you have done ſetting down all your Dimenſions, compare your Dimenſions together, to ſee if the Dimenſions in both your Books agree.

The Reaſon why you ſet down the Product of each Dimenſion juſt againſt it in the next Column towards the Right Hand, is,

If you meaſure againſt another Meaſurer, and there ſhould be a Miſtake in either of your caſtings up of the Dimenſions (as it often happens thro' Security or

Negligence) then one by reading over the Dimenſions in his Book, with the Product to each Dimenſion as he goes on; and the other looking in his own Book, the Miſtake will ſoon be found, which muſt be rectified between you.

Therefore, to be certain in caſting up your Dimenſions, you ought to caſt them up twice, if not three times; (to wit) after you have caſt them all over once, begin and caſt them over again, and ſee whether it agrees with your firſt caſting up; if not, then caſt them up again.

When you make your Bill of Meaſurement, you muſt ſet your Name to it at the lower End of the Bill.

An Example of a Bill.

Glaziers Work done by *A. B*, for *C. D*, in *Long-Acre*, and meaſured the 22d of *January*, 1727.

	l.	*s.*	*d.*
For 96 Feet, and 3 Inches of Glazing done with Quarries, at 5d. *per* Foot	2	00	1 $\frac{1}{4}$
For 52 Feet, and 1 Inch of Glazing with Squares, at 7d *per* Foot	1	10	4 $\frac{1}{2}$
The Sum is	3	10	5 $\frac{3}{4}$

Meaſured the Day and Year above-written by V. M.

You may ſometimes happen to meet with Panes of Glaſs of various Forms; as ſometimes the Top of a Pane of Glaſs is concluded with a Semicircle, ſometimes

times with a Scheme or Segment of a Circle, ſometimes Triangular, and ſometimes Ovalar, and ſometimes an Oval in Glaſs, or a round Pane.

For the Meaſuring of each of theſe, and ſuch like, I refer you to the Meaſuring of Planes in the Fifth Book, which doth ſucceed this Treatiſe.

Thus much may ſuffice the ingenious Practicer, as to the Meaſuring of Glazing.

CHAP. V.

THE Meaſuring of Joiners Work is the ſame that is taught before, only there is this Difference; Carpenters Work is brought into Squares, and Glaziers into Feet; and Joiners Work muſt be brought into Yards.

Which to do, after you have caſt up all your Dimenſions, and brought them into Feet, divide the whole Product by 9, and that gives you the Number of Yards.

You muſt divide by 9, becauſe 3 Feet in Length, and 3 in Breadth, contain 9 Feet, which is a ſuperficial Yard.

I will ſuppoſe ſome Dimenſions to be caſt up, whoſe Contents or Products are theſe following.

For it would be needleſs to ſpend time to teach the caſting up of Dimenſions in Feet and Inches again, having ſpoken largely and plainly thereto in the precedent Pages.

Wainſcot.

Feet.	Inches.
110	08
304	07
065	06
934	02
041	00
073	01
1529	00

68 (8
1529 (169 Yards, and 8 Feet.
999

The Total; which divide by 9 to bring the Feet into Yards, as you ſee above, and in 1529 Feet, you find 169 Yards, and 8 Feet;

Which muſt be ſet down in the Bill of Meaſurement at ſo much *per* Yard.

Then, for Cornices, and Baſe, and Sub-baſe, Joiners do them by the Foot, Line Meaſure, or Running Meaſure, as it is called, if they do no other Work with it: But if they do other Work joining with it, they meaſure it all together by the Yard.

Likewiſe Architrave and Frieze they do by the Foot Line Meaſure.

The Chimney-Pieces at ſo much a-piece, and ſometimes meaſure them in to the other Work; deducting nothing for the Vacancy of the Chimney.

CHAP.

CHAP. VI.

PAinters Work is meaſured like Joiners Work, and brought into Yards, only there is this Difference:

When you meaſure Painting upon Cornices, or Moldings of any Sort, you muſt have a Line, and girt the round Moldings, and bend the Line into the hollow Moldings. Thus when you are to meaſure a Room, you begin with your Line at the Top of the Cornice, where you may faſten it with a ſmall Nail, and girting the round Moldings, and bending the Line into the Hollows and Angles, bring the Line down to the Bottom of the Painting, then meaſure the Length of the Line with your Rule, and ſet down that Sum.

Then begin in one Angle of the Room, and meaſure the Length of that Side whereof you took the Height, and ſet down the Length under the Height which before you ſet down, and that is one Dimenſion.

And if the oppoſite Side of the Room be like unto that which you have already meaſured and ſet down, and the Room ſquare, you may either ſet down that Dimenſion twice, or elſe ſet a Figure of (2) encloſed againſt the Dimenſion, which ſignifies that Dimenſion is 2 times: Alſo you muſt take the Length of the other 2 Sides, and ſet them down by the ſame perpendicular Height; or you may take the Length of each Side of the Room, and add them all together, and ſo make but one Dimenſion of all the Sides by the perpendicular Height.

Then after you have ſet down all your Dimenſions, and brought them into Feet and Inches by caſting them up, you muſt divide the whole Product by 9, and that brings the Feet into Yards, as you read before: For you muſt not think to learn to meaſure any one of theſe Trades only, but you muſt begin at the beginning of Menſuration (to wit) of Carpenters Work, and ſo proceed gradually; and be ſure to be perfect in caſting up one Dimenſion before you enter upon another.

Thus proceeding with Care and Diligence, until you have attained to the meaſuring of Glazing, you may with eaſe afterwards meaſure all the reſt of the Works that are ſuperficial.

Indeed, the meaſuring of Bricklayers Work is ſomething more difficult, becauſe it is reduced to a thickneſs, and ſo becomes ſolid Meaſure.

As for the Painting of Windows, they are generally ſet down at ſo much *per* Light, and Caſements at ſo much a-piece.

The next Work to be meaſured is Plaiſtering, of which I need ſay no more than this, that it muſt be brought into Yards; and where a Sommer or Girder lies below the Ceiling, it is uſually deducted where the Workman finds Materials; but not elſe.

Alſo in rendring where Materials are found by the Workman, ſometimes ¼ is deducted for the Quarters; but not where Workmanſhip only is found, becauſe it might be rendred as ſoon if there were no Quarters.

The meaſuring of Maſon's Work (to wit, flat Paving, and ſuch like) is meaſured the ſame Way as all the reſt. There is no need of Diviſion, for Maſon's Work by the Foot.

I come now to treat of the Menſuration of Bricklayers Work.

CHAP.

CHAP. VII.

BRickwork, as you read before, muſt be reduced to a certain or uſual Thickneſs, in caſting of it up, for which Reaſon it becomes ſolid Meaſure.

Which uſual Thickneſs is 14 Inches, or, as it is generally called, one Brick and half.

By one Brick and half, is meant the Length of a Brick, and half the Length; or the Length of one Brick, together with the Breadth of another.

For the Breadth of two Bricks, with a Joint of Mortar between them, is anſwerable to the Length of a Brick, the Length whereof ought to be, and generally is 9 Inches, and the Breadth 4 Inches and $\frac{1}{4}$, which being added together with a Joint of Mortar makes 14 Inches, the aforeſaid uſual Thickneſs.

And as Carpenters Work is brought into Squares, Bricklayers Work muſt be brought into Rods, which Rod contains 272 Feet and 3 Inches, and is produc'd from 16 Feet and 6 Inches in Length, being multiplied by 16 Feet and 6 Inches in Height or Breadth; or a Quadrate whoſe *Side* is a Pole or Perch.

When you come to meaſure the Brickwork of a Building, you will find the Walls thereof to be of ſeveral Thickneſſes; ſome thicker than a Brick and half, and ſome thinner, as 1 Brick Walls, or Walls 9 Inches in Thickneſs.

Which ſeveral Thickneſſes muſt all be brought, or reduced to the uſual Thickneſs of 1 Brick and half, which to do, obſerve the following Rules.

1. Set down each Thickneſs in a Column by it ſelf, (to wit) ſet down your Dimenſions that are

1 Brick (or 9 Inches) in Thickneſs, in a Column by themſelves.

Alſo thoſe Dimenſions that are 1 Brick and ½ in Thickneſs, muſt be ſet in a Column by themſelves.

Likewiſe thoſe of 2 Bricks in Thickneſs by themſelves, and thoſe of 2 Bricks and ½, in a Column by themſelves.

And thoſe of 3 Bricks in a Column by themſelves.

So likewiſe if your Walls be thicker than 3 Bricks, (in Length) ſet each Thickneſs by it ſelf.

By ſetting the Dimenſions by themſelves, is meant to ſet them in a Page or Column alone ſeverally, and not to intermix Dimenſions of ſeveral Thickneſſes in one Page or Column.

Notwithſtanding, ſome Meaſurers ſet down their Dimenſions as they take them of ſeveral Thickneſſes in one Column; but I do not think it ſo convenient for a Learner, as to ſet them down ſeverally.

2. After you have taken your Dimenſions and caſt them up, and by adding the ſeveral Products together (of one Thickneſs) have produced the total Product, then to reduce it to 1 Brick and ½, work thus;

If the Dimenſions that you have caſt up, be 1 Brick's Length in Thickneſs, that is the Breadth of 2 Bricks; then you muſt multiply your total Product by two, which is the Thickneſs of the Wall in the leaſt Denomination (to wit) two 4 Inches, or the Breadth of 2 Bricks, and divide that Product by 3, (becauſe there is the Breadth of 3 Bricks in a Brick and ½) and the Quotient is the Sum of the 1 Brickwork, reduced to 1 Brick and ½ Brickwork.

Example.

Suppoſe you have caſt up ſeveral Dimenſions of 1 Brick's Length in Thickneſs, and the Products (or Contents) of each Dimenſion being added together, produceth a total Product of 4976 Feet of Brick-work, being one Brick thick.

To reduce this to 1 Brick and ½ thick, multiply 4976 by 2 (to wit, the Breadth of 2 Bricks, and that brings it into the leaſt Thickneſs or Denomination, namely, 4 Inch-work, or Brick-work ½ a Brick thick.)

$$\begin{array}{lr} & 4976 \\ & 2 \\ \hline \text{So the Product of 4 Inch-work is} & 9952 \\ \hline \end{array}$$

Which divide by 3 (becauſe there are the Breadth of 3 Bricks in 1 Brick and ½) and in the Quotient you have 3317 Feet, of 1 Brick and ½ Work, and 1 remaining.

$$\begin{array}{lr} & \cancel{2}1 \\ & \cancel{9}\cancel{9}\cancel{5}\cancel{2}\ (3317 \\ \text{See the Diviſion} & \cancel{3}\cancel{3}\cancel{3}\cancel{3} \end{array}$$

The 1 that remains is 1 Foot of 4 Inch-work, or 4 Inches of 1 Brick and ½ Work being reduced.

For if you divide 12 (which are the Inches in that Foot) by 3 (the Number of 4 Inches or ½ Bricks in a Brick and ½) the Quotient will be 4, which is 4 Inches or ⅓ of a Foot.

So likewiſe your Dimenſions that are thicker than 1 Brick and ½) as ſuppoſe 2 Bricks in Length, which is called 2 Brick-work, which 2 Brick-work contains the Breadth of 4 Bricks, you muſt in reducing 2 Brickwork to 1 Brick and ½ work, firſt multiply the Sum by 4, and divide the Product by 3.

Example.

If you have 5845 Feet of 2 Brickwork, multiply it by 4.

```
See the Multiplication          5845
                                   4
                               ——————
        The Product is          23380 Feet
                               ——————
```

Of 4 Inch Work, which you muſt divide by 3.

```
                                 221 1
See the Diviſion.               23380(7793
                                 3333
```

And the Quotient is 7793 Feet of 1 Brick and ½ Work, and 1 remaining, which is 1 Foot of 4 Inch Work, which being reduced, as you are taught in the foregoing Page, is 4 Inches (or ⅓ of a Foot) of 1 Brick and ½ Work. Thus in 5845 Feet of Brickwork being 2 Bricks thick (or the Walls being the Length of 2 Bricks in Thickneſs) you have, when it is reduced, 7793 Feet, and 4 Inches of 1 Brick and ½ Work, (the uſual Thickneſs.)

If your Dimenſions be 2 Bricks and ½ in Thickneſs, you muſt multiply the total Product by 5, and divide the Product of that Multiplication by 3.

If your Dimenſions be 3 Bricks in Thickneſs, you may ſave the Labour of dividing and multiplying, becauſe a Wall of 3 Bricks thick is juſt ſo much more as a Wall of 1 Brick and ½ thick; therefore if you ſet down the Sum of your Product in 3 Bricks twice, it will be the ſame as if you ſhould multiply by 6, and divide the Product by 3, for this Reaſon, that 6 is as much more as 3.

In meaſuring of Brickwork you muſt ſet down your Dimenſions with the Number of their Thickneſſes over them, or adjoining to them; namely, over Dimenſions of 1 Brick in Thickneſs ſet 1 B. ſignifying 1 Brick; and over Dimenſions that are 1 Brick and ½ in Thickneſs, write ½ B, and over Dimenſions 2 Bricks in Thickneſs, write 2 B, &c.

And caſt up your Dimenſions as you are taught in meaſuring of Carpenters Work and Glaziers Work.

But in Brickwork as well as in Carpenters Work, it is uſual to meaſure no nearer than whole Inches, that is to ſay, ½ Inches and ¼ of Inches, nor ¾ of Inches are not ſet down.

For which Parts of Inches an Allowance is made when you come to divide your Number of Feet to bring them into Rods.

To bring or reduce Feet into Rods.

After you have reduced all your ſeveral Thickneſſes, and brought them to the uſual Thickneſs of a Brick and ½, you muſt divide your whole Product by 272, (being the Number of Feet contained in a Rod) to bring or reduce the Feet into Rods.

Taking no notice of the 3 Inches belonging to the 272 Feet, being both contained in a Rod.

For you read before, that in a Rod is contained 272 Feet, and 3 Inches, which 3 Inches is allowed to make good for the Parts of Inches, which is not ſet down when you ſet down the Dimenſions.

But if you are minded to be exact and curious in meaſuring, you may ſet down the odd Parts of Inches, and caſt them up as you were taught in meaſuring of Glazing, and divide the total Product by 272 Feet and 3 Inches, but this is ſeldom or never done in Brick-work.

Folded between Page 240, 241, you have deſigned the Ground-plat of a Building; which together with the Inſtructions that follow, will much aſſiſt you in making an Eſtimate for Building from a Deſign given.

It will alſo inſtruct you how to take your Dimenſions, and to ſet them down in your Book, better than by a Multitude of Words.

The Explanation of the Deſign.

This Deſign is the Ground-plat of a Building, being 25 Feet in the Front, and 40 Feet in the Flank or Depth.

The Front and Rear-Front Walls are 2 Bricks and $\frac{1}{2}$ thick.

The Flank Walls are 2 Bricks thick, as you may perceive by the Scale annexed to the Deſign.

You may ſuppoſe this Deſign to be the Ground Floor, having no Cellar beneath it.

And the Story to be 11 Feet and 6 Inches in Height from the Foundation to the Top of the next Floor.

The Windows are 4 Feet in Breadth, and 6 Feet 6 Inches in Height or Length.

The Door-caſes in Front and Rear are 4 Feet wide, and 9 Feet in (Length or) Height.

The Chimneys, you ſee, ſtand back to back, the Jaums being 2 Bricks thick.

The Encloſure of the Stairs, and the Partition next the Entry, are of Timber.

What hath been ſaid is ſufficient for Explanation.

I ſhall, in the next place, inſtruct you how to meaſure the Brickwork of this Story in the Ground-plat, which being well underſtood, you may with eaſe meaſure all the reſt of the Stories over it.

I ſhall begin with the Front.

And becauſe all Walls are (or ſhould be) made with a Baſis, therefore we will ſuppoſe the Front Wall and Rear Front-Wall to be 3 Bricks thick, 6 Inches high, which is 2 Courſes of Bricks; for 3 Inches is generally allow'd for a Courſe (or the Thickneſs of a Brick laid in Mortar.

So the firſt Dimenſion will be 25 Feet in Length, by 6 Inches in Height, 3 Bricks thick.

The Thickneſſes muſt be always ſet over the Dimenſions (as you will ſee in the next Page.)

And becauſe the Front and Rear are both of one Length, you need not ſet down the Dimenſion twice, but ſet a Figure 2 encloſed in a Circle thus (2) which ſignifies the Dimenſion to be 2 times.

I think it will be beſt for Inſtruction, to caſt up the Dimenſions as they are taken, (or ſet down) but after you underſtand the Way, you muſt ſet down all your Dimenſions firſt, and caſt them up afterwards.

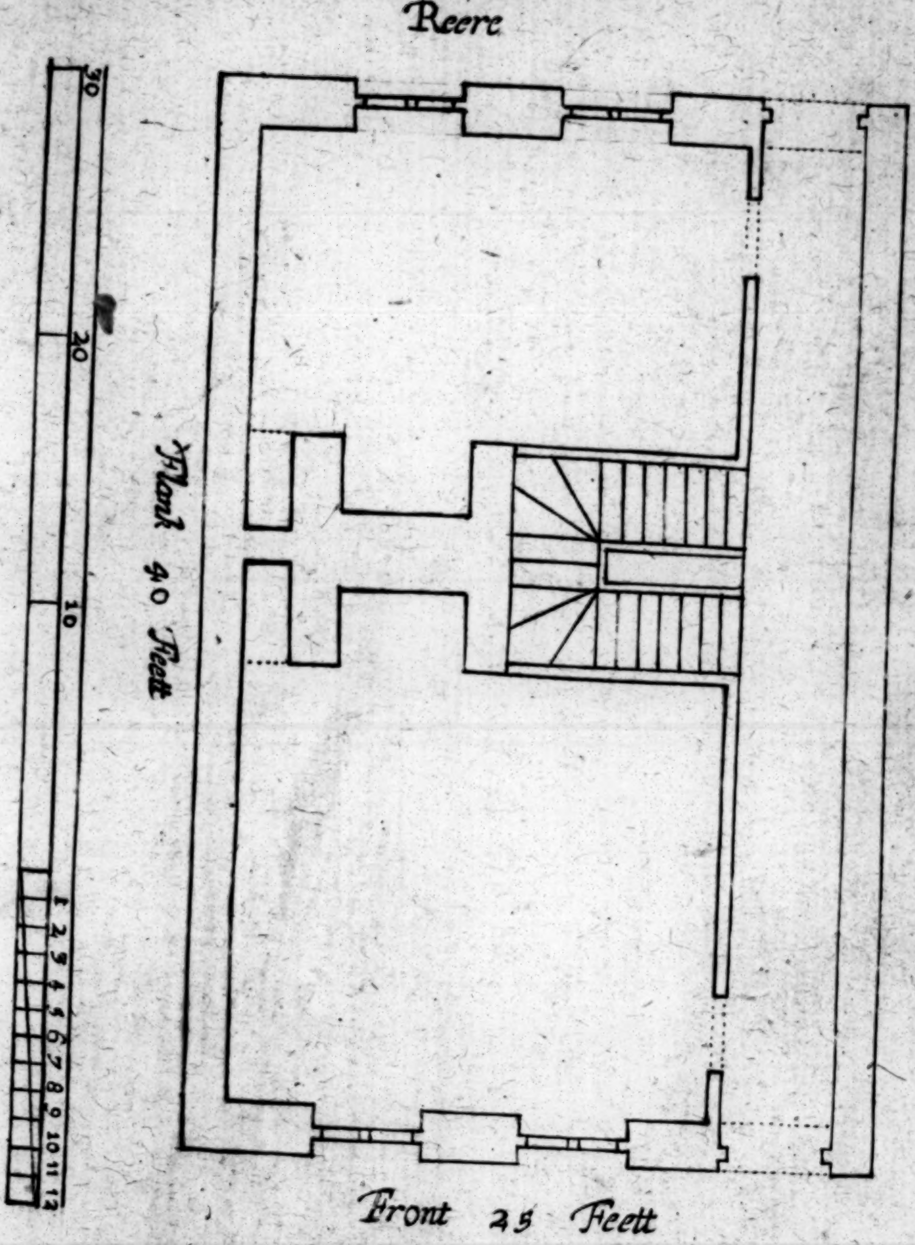
I ſhall alſo ſet the ſeveral Thickneſſes of the Dimenſions (of this Story of the Ground-plat) in one Column, becauſe there will be but a few Dimenſions in this Story, and not above a Dimenſion or two of one Thickneſs, and it is not worth while to ſet down a Dimenſion or two in a Column by it ſelf.

Example.

	3 B.		
	Feet.	In.	
The Length of the Front is	25	00	(2)
The Height of the Brickwork is 2 Courſes, or	00	06	
The Product is	12	06	
The Rear Front being the ſame Dimenſion, and being joined in one Dimenſion by the Figure (2) you muſt add	12	06	
Which being added, the Content of the 3 Brickwork is	25	00	
And becauſe 3 Bricks is the Double of 1½ B. add	25	00	
	1½ B.		
And the Content of this firſt Dimenſion being reduced into 1½ B. is	50	00	
	2½ B.		
2dly. I proceed to the remaining Part of the Front and Rear, being in Height	11	00	(2)
In Length	25	00	
Being multiplied	55		
	22		
The Content of the Front is	275	00	
The ſame being added for the Rear	275	00	
The Content of 2½ B. work in Front and Rear is	550	00	
Which to reduce, you muſt multiply by	5		
The Prod. of 4 Inch. or ½ B. work is	2750		

Which

Betwen Pag. 240.& 241.



Which muſt be divided by 3, to reduce it to 1 ½ B. in Thickneſs.

$\cancel{2}2$

$\cancel{2}\cancel{7}\cancel{5}\cancel{0}(916$

$\cancel{3}\cancel{3}\cancel{3}$

And the Quotient is 916 and 2 remaining, which is 916 Feet of 1 ½ B. and 2 Feet of ½ B. which is 8 Inches of 1 ½ B. being reduced.

Having done with the Fronts, we proceed to take the Dimenſions of the Flank-Walls.

And becauſe we meaſured the Fronts from Out to Out, as we call it in brief, (to wit, from the Outſide of one Flank-wall, to the Outſide of the other) we muſt meaſure the Length of the Flank-walls within (to wit, from the Inſide of the Front-wall, to the Inſide of the Rear Front-wall) ſo the Thickneſs of the two Front-walls, which is 3 Feet and 10 Inches, being taken out of the 40 Feet in Depth,

	½ B.		
	Feet.	Inches	
The Length of one Flank-wall is	36	02	(2)
The Height of the Baſe is	00	06	
And becauſe both Walls are alike, or equal in Length and Height, you muſt put (2) againſt the Dimenſion, and it ſerves for both Walls	18 18	01 01	
The Cont. in 2 ½ B. work of both Walls is	36	02	
Being multiplied by	5		
The Product of ½ B. work is	180	10	

Being divided by 3 to reduce to ½ B. $\cancel{1}8\cancel{0}(60\ 1$

$\cancel{3}\cancel{3}\qquad \cancel{1}\cancel{0}(3$

$\cancel{3}$

It is 60 Feet, 3 Inches and ½ of an Inch of 1 ½. B. Work.

The

	2	B.	
The next Dimenſion of the Flank-wall is	36	02	(2)
	11	00	
	36		
Being multiplied	361	10	
The Cont. in 2 B. work of one Wall is	397	10	
To which add the other Wall	397	10	
The Cont. in 2 B. work of both Walls is	795	08	
Which multiply by		04	
The Con. of both Walls in ½ B. work	3182	08	

Being reduced to 1 ⅓ B. the Content of both Walls are 1060 Feet and 10 Inches.

3182(1060
3333

Although I caſt up and reduce each Dimenſion as I proceed, yet after you underſtand the Way, you muſt reduce all your Dimenſions of one Thickneſs at one time.

Having caſt up and reduced both the Fronts, and both the Flank-walls, the next Dimenſions to be taken are the Chimneys: Which are uſually agreed for, at ſo much *per* Fire-place (or Chimney). Alſo ſometimes they are done by the Rod, at the Rate of the other Work, and then they muſt be meaſured.

Theſe two Chimneys in the Deſign, ſtanding Back to Back, are 5 Feet a-piece between the Jaums.

The Jaums are 2 Bricks thick.

The Wall between the Chimneys is 14 Inches thick; to which is added 9 Inches, for a falling Back to each Chimney.

Therefore we will take the Wall firſt by it ſelf, which is 9 Feet 10 Inches in Length, and 11 Feet 6 Inches, the aforeſaid Height of the Story, and ſet it down.

	1 ½ B.	
The Heighth	11	06
The Length	09	10
	99	
	4	06
	9	02
		05
The Cont. of the Wall between the Chim.	113	01

Altho' heretofore I have put Feet or F. over the place of Feet; and In. or I. over the Place of Inches. Suppoſing now that you know one Place from another, I ſhall deſiſt doing it, as in the laſt Dimenſion; and ouly ſet the Thickneſſes over the Dimenſions.

In the next place, ſuppoſe the Mantle-tree to lie 5 Feet high, and the falling back to begin at 3 Feet high The Mean between 5 and 3 is 4, which is the Height of the 9 Inch Work that is added to each Chimney for falling back, and the Length is 5 Feet.

	1 B.		(2)
The Dimenſion of the falling back of both Chimneys	05	00	
	04	00	
	20	00	
	20	00	
The Content in 1 B.	40	00	

Multi-

	1	B.
	40	00
Multiply it by	2	
The Product is	80	
	22	
Which divide by 3	~~80~~	(26
	~~33~~	

Having reduced it, you find 26 Feet and 8 Inches of 1 ½ B. work in both fallings back.

The 4 Jaums being 3 Feet 6 Inches deep apiece, above the falling back; being added together, make 14 Feet of 2 B work, which must be multiplied by the Height of the Story.

For although the Jaums in the next Story, be but 1 ½ B. and from the Mantle-tree in this Story it be wrought but a Brick and half, yet the Wings being added, makes two Bricks.

	2	B.
The Breadth of the 4 Jaums being added together is	14	00
The Height of the Story is	11	06
	14	
	147	
The Cont. of the 4 Jaums in 2 B. work is	161	00
Being multiplied by	4	
The Cont. of the 4 Jaums in ½ B. work is	644	
	12	
Which divide by 3 in the Margin	~~644~~	(214
	~~333~~	

And the Content of the Jaums being reduced to 1 ½ B. is 214 Feet, and ⅔ of a Foot, or 8 Inches.

The 2 Breaſts of the Chimneys are next to be meaſured, which are 5 Feet apiece between the Jaums; and altho' they are but 9 Inches thick apiece, yet conſidering the croſs Withs, and the Peers that are wrought from the falling back till the Wings gather to them, they are uſually meaſured at the ſame Thickneſs that the Jaums are, and at the Height of the Story, abating nothing for the Vacancy between the Floor and the Mantle-tree.

	2 B.		
The Length of the Breaſt between the Jaums is	05	00	(2)
The Height of the Story is	11	06	
	05		
	52	06	
	57	06	
The Dimenſion being (2) add	57	06	
The Content of the 2 B. work in both Breaſts is	115	00	
Which multiply by	4		
Being brought into the leaſt Denomination, or ½ B. it is	460	00	
	$\cancel{1}\cancel{1}\cancel{1}$		
Which divide by 3	$4\cancel{6}\cancel{0}$(153		
	$\cancel{3}\cancel{3}\cancel{3}$		

Being reduced, the Content is 153 Feet and ⅓ of a Foot (or 4 Inches) of 1 ½ B.

Thus having taken all the Dimenſions in one Story, and caſt them up, The next Thing to be done, is to take the Deductions of the Windows and Doors in the Fronts.

Note, That the Deductions (to wit, Windows and Doors, and all Vacancies whatſoever that you meaſure) muſt be ſet in Columns by themſelves, with this Note *Ded.* (ſignifying Deductions) over them, together with the Thickneſs of the Wall wherein the Windows or Doors ſtand that you are to deduct.

Example.

	Ded.	2 ½ B.	
The Height or Length of a Window is	06	06	} (4)
The Breadth is	04	00	}

And becauſe there are 4 Windows in both Fronts, and all of one Bigneſs; therefore put (4) againſt the Dimenſion, ſignifying that *that* Dimenſion is 4 times (or that there are 4 Windows of that Bigneſs.)

	Ded	2 ½ B.	
The Height of a Window	06	06	} (4)
The Breadth	04	00	}
	24	00	
Being multiplied	2		
The Prod. of 2 ½ B. work in 1 Win. is	26	00	
Which multiply by	4		and
the Prod. of 2 ½ B. work in the 4 Win. is	104		
Which multiply by	5		
To bring it into the leaſt Denomination, and it is	520		

Which divide by 3; and Being reduced to 1 ½ B. there is 173 Feet and 4 Inches.

$\cancel{2}\cancel{1}1$
$\cancel{5}\cancel{2}\cancel{0}$ (173
$\cancel{3}\cancel{3}\cancel{3}$

There are alſo in Front and Rear 2 Door-caſes to be deducted (deduct ſignifies to take out) of the ſame Thickneſs of 2 ½ B.

	Ded.	2 ½	
The Height of a Door is	09	00	(2)
The Breadth	04	00	
Being multiplied	36		
	2		
The Product of 2 ½ B. work in both Doors is	72		
Which multiply by	5		
Being brought into ½ B. work is	360		
Which divide by 3, as in the Margin	$\cancel{3}\cancel{6}\cancel{0}$ (120		
	$\cancel{3}\cancel{3}\cancel{3}$		

being reduced to 1 ½ B. there is 120 Feet, as in the Margin.

Note, Tho' I reduce the Deductions here for Inſtruction; hereafter you need not reduce them, but take the Product of the Deductions of one Thickneſs out of the Product of Dimenſions of the ſame Thickneſs.

In the following Page I will ſet down all the Dimenſions as they were taken, with the Product of each Dimenſion in a Column juſt againſt it.

And although I ſaid before, that you might divide a Page or Leaf of your Meaſuring Book into 4 Parts or Columns; yet in Meaſuring of Bricklayers Work, it will be neceſſary to divide a Page only in two Parts, (as you ſee in the next following Page) that ſo you may have room to ſet a Name to each Dimenſion for Diſtinction-ſake.

Dimenſions of the Deſign.			*Products.*
Baſis of the Front and Rear	3 B. 25 00 06	(2)	3 B. 25 00
Front and Rear	2 ½ B. 25 00 11 00	(2)	2 ½ B. 550 00
Baſis of both the Flank Walls	2 ½ B. 36 02 06	(2)	2 ½ B. 36 02
Both the Flank Walls	2 B. 36 02 11 00	(2)	2 B. 795 08
The Wall between the Chimney	½ B. 11 06 09 10		1 ½ B. 112 01
The falling back of both Chimneys	1 B. 05 00 04 00	(2)	1 B. 040 00
The 4 Jaums	2 B. 14 00 11 06		2 B. 161 00
The Forepart or Breaſts of both Chimneys	2 B. 11 06 05 00	(2)	2 B. 115 00

Having ſet down all the Dimenſions with their Products, in the next place we muſt ſet down the Deductions of the Windows and Doors with their Products.

Deductions

Deductions in the Deſign.	*Ded.*		*Products.*
	2 ½ B.		
The 4 Windows	06 06 04 00	(4)	2 ½ B. 104 0
	2 ½ B.		
The 2 Doors	09 00 04 00	(2)	2 ½ B. 072 0

The next Work is to add the Products of each ſeveral Thickneſs into one Sum.

Products of ſeveral Thickneſſes.

3 B.	2 ½ B.	2 B.	1 ½ B.	1 B.
25 00	550 00	795 08	113 1	40 0
	36 02	161 00		
	586 02	115 00		
		1071 00		

The ſeveral Products of each Thickneſs being added,

In the firſt Column of the left Hand, you have 25 Feet of 3 B.
In the ſecond Column you have 586. 2. of 2 ½ B.
In the third Column you have 1071. 8. of 2 B.
In the fourth Column you have 113. 1. of 1 ½ B.
In the fifth Column you have 40 Feet of 1 B.

The next Work muſt be to take the Products of the Deductions out of the Products of the Dimenſions.

Products of Ded. in 2 ½ B.
104 0
72 0

The total Product of Ded. in 2 ½ B. is 176 0

Which 176 Feet of 2 ½ B. work being contained in the Windows and Doors, muſt be ſubtracted from 586 Feet and 2 Inches, being the total Product of all the Dimenſions that are 2 ½ B. in Thickneſs.

For this Reaſon.

Becauſe when we meaſured the Front and Rear, we meaſured the whole Length and Breadth over the Windows and Doors, allowing no Abatement for them.

Note, That whatſoever Windows or Doors, or other Vacancies you meaſure over when you take the Dimenſions, you muſt remember to deduct them out of the total Product of the Dimenſions of the ſame Thickneſs wherein they are ſituate.

Example.

The Doors and Windows being in 2 ½ B. work; I ſet down the total Product of all the Dimenſions of that Thickneſs, which is } 586 02
The total Product of all the Deductions of that Thickneſs which are to be ſubtracted is } 176 00
The Remainder is 410 02

So likewiſe if there had been Deductions in the other Thickneſſes, you muſt have ſubtracted them before you begin to reduce your ſeveral Thickneſſes to the uſual Thickneſs of 1 ½ B.

But ſeeing we have no other Ded. in this Deſign, to ſubtract out of any other Thickneſs, the next Work will be to reduce each Thickneſs to the uſual Thickneſs of 1 ½ B.

Beginning with the greateſt Thickneſs (to wit 3 B.) we find 25 Feet, which, as you read before, becauſe it is juſt as thick again as 1 ½ B. you need not multiply it by 6, and

Products reduced.

1 ½ B.

divide it by 3, but add juſt ſo much more to the Dimenſion as it is (to wit 25 to 25) and ſo you will have 50 Feet, which muſt be ſet down in a Column with 1 ½ B. over it. 0050 0

The next Thickneſs to be reduced is 2 ½ B. the total Product whereof is (the Deductions being taken out) 1410 Feet and 2 Inches, which multiply by 5, and divide the Product by 3.

Example.

```
        2 ½ B.
   410   2            211
         5            2050  (683
½ B. Work 2050 10     333
```

The 1 that remains in Diviſion is 4 Inches, and the 10 Inch. in the ½ B. work is 3 Inches of 1 ½ B. work, which being added is 7 Inches; ſo the 2 ½ B. work being reduced is 683. 7. which muſt be ſet in the Column. 0683 7

```
The next Thickneſs to be reduced is       2 B.
2 B, the total Product whereof is     1071   8
     Which multiply by                       4
And the 4 Inch (or ½ B.) work is      4286   8
```

```
The 2 remaining in Diviſion  }  1022
is 8 Inches, and the 8 Inches of }  4286 (1428
½ B. is 2 Inches of ½ B. which }  3333
```

added is 10 Inches, ſo the 2 B. work being reduced is 1428 10

The next Thickneſs being 1 ½ B. needs no reducing, becauſe it is the uſual Thickneſs, the Product whereof is 0113 1

```
The next Thickneſs to be reduced is       1 B.
1 B. the total Product whereof is           40
     Which multiply by                       2
And the Product of ½ B. work is             80
```

```
22        Which divide by 3, and the Quotient is
80 (26    26 Feet and 8 Inches, being reduced;      0026  8
33
```

The Product of all the Thickneſſes being reduced is 2302 2

Having reduced all the Thickneſſes to 1 $\frac{1}{2}$ B. in Thickneſs, you find the total Product to be 2302 Feet and 2 Inches.

Which 2302 muſt be divided by 272, the Number of Feet contained in a Rod, as you read before.

Example.

$$\begin{array}{r} 126 \\ \cancel{2}\cancel{3}\cancel{0}\cancel{2}(8 \\ \cancel{2}\cancel{7}\cancel{2} \end{array}$$

Being divided, the Quotient is 8, and the Remainder is 126, which is 8 Rods and 126 Feet and 2 Inches (if you add the two Inches that belonged to the 2302 Feet which you divided.)

Note, The whole Rod containing 272 Feet, the half Rod contains 136 Feet, and the Quarter of a Rod contains 68 Feet; for twice 136 is 272, and four times 68 is the ſame.

Then take 68 Feet, or $\frac{1}{4}$ of a Rod, out of the 126 Feet that remain, by ſubtracting 68 from 126

68

And the Remainder is 58 Feet

So there is 8 Rods and $\frac{1}{4}$ and 58 Feet of 1 $\frac{1}{2}$ B work in one Story of the Deſign.

It is convenient, before I proceed farther, to add ſomething more concerning meaſuring of Chimneys.

Firſt *Note*, If you are to meaſure a Chimney ſtanding alone or by it ſelf; without any Party-wall being adjoyned;

Then girt it about for the Length, and the Height of the Story is the Breadth.

The Thickneſs muſt be the ſame that your Jaums be, provided the Chimney be wrought upright from the Mantle-tree to the Ceiling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, becauſe of the Gatherings of the Breaſt and Wings to make room for the Hearth in the next Story.

Secondly Note, If the Chimney-back be a Party-wall, and the Wall be meaſured by it ſelf, then you muſt meaſure the Depth of the 2 Jaums, and the Length of the Breaſt, which being added together, will be your Length, and the Height of the Story your Breadth, at the ſame Thickneſs your Jaums be.

Thirdly Note, When Chimneys are agreed for at a Rate by the Fire-place or Chimney-building, and the Back of the Chimneys ſtand againſt a Wall (that you meaſure beſides) you muſt deduct 4 Inches in Thickneſs out of the Menſuration of the Wall againſt the Chimneys, which 4 Inches is allowed for the Backs of the Chimneys.

Fourthly Note, When you meaſure Shafts of Chimneys, girt them with a Line round about the leaſt Place of them for the Length, and their Height ſhall be your Breadth; and if they be 4 Inch-work, then you muſt ſet down your Thickneſs at 1 B. work: But if they be wrought 9 Inches thick (as ſometimes they are when they ſtand high and alone above the Roof) then you muſt account your Thickneſs 1 $\frac{1}{2}$ B. in Conſideration of Withs and Pargetting, and Trouble in Scaffolding.

Obſerve theſe following Rules, which will much help you in caſting up your Dimenſions of any Kind, and ſave you the Labour of Diviſion, to know how many Feet there is in any Number of Inches that you multiply into Feet.

Let the firſt Example be

Firſt, Having multiplied the whole Feet, in the next place you multiply the Inches into the Feet, ſaying 2 times 18 is 36 Inches; or thus, 18 Inches being a Foot and an half, ſay, 2 Feet and 2 half Feet are 3 Feet, which being ſet down:

30	02
18	03
240	
303	
7	06
550	06½

In the next Place you come to multiply 3 into 30, which you need not multiply; for 3 being one fourth Part of 12, ſay, The one ¼ of 30 Feet is 7 Feet and 6 Inches; which is a quicker Way than to multiply 30 by 3, and divide the Product by 12.

Let the ſecond Example be

The Feet being multiplied and ſet down, next you are to multiply 4 by 42; but 4 being ⅓ Part of 12, ſay, the one Third of 4 is 1, and the one Third of 12 is 4, which is (being added) 14 Feet.

In the next Place you come to multiply 5 25, which you may ſay is 25 half Feet lacking 25 Inches, that is, 10 Feet and 5 Inches.

25	04
42	05
50	
100	
14	
10	5
	1
1074	6

Or thus, 24 Inches being 2 Feet, ſay 5 times 24 Inches is 10 Feet, then add the 5 Inches, it makes 10 Feet and 5 Inches. Laſtly, 5 by 4 is 20, which is 1 Inch and 8 Parts, which 8 Parts is not accounted, nor ſet down, (as you have read before.)

Let the third Example be

100	7
94	6
400	
900	
50	
47	
7	10
	3
9505	1

The Feet being multiplied, you proceed to multiply 6 by 100, but 6 being the half of 12, ſay the half of 100 is 50 Feet, which being writ down, you proceed to multiply 7 into 94, then ſay the half of 94 is 47 Feet, which being ſet down, there is ſtill remaining 94 Inches, which you may ask your ſelf how many times 12 you can have in 94? Your Anſwer will be 7 times, and 10 Inches remaining, therefore you muſt ſet down 7 Feet and 10 Inches. Laſtly, you proceed to multiply the Inches by the Inches, ſaying 7 times 6 is 42, which is 3 Inches and ½; ſo the total Product is 9505 Feet and 1 Inch, as you ſee in the Margin above.

Let the fourth Example be

230	8
310	9
2300	
690	
206	8
115	
57	6
	6
71679	8

The Feet being multiplied and ſet down, you proceed to multiply 310 by 8; but inſtead of multiplying, take the ⅔ of 310, becauſe 8 is ⅔ of 12; ſaying thus, the ⅔ of 3 is 2, but the ⅔ of 1 you cannot take in Integers, therefore you add a Cypher next to the Figure 2, and proceed to take the ⅔ of 10, which is 6 Feet and 8 Inches; ſo you find the $\frac{2}{3}$ of 310 Feet to be 206 Feet and 8 Inches; which being ſet down,

The next is 230 Feet to be multiplied by 9 Inches; which 9 Inches is $\frac{3}{4}$ of 12 Inches; therefore you muſt take $\frac{3}{4}$ of 230 Feet, ſaying, the half of 230 is 115, and $\frac{1}{2}$ of 115 is 57 Feet and 6 Inches, which being added to 115 make 172 Feet and 6 Inches, as you ſee by caſting up the Dimenſion,

Let the fifth Example be

94	11
48	10
752	
376	
47	
31	4
44	9
4635	1

The Feet being multiplied and ſet down, inſtead of multiplying 10 by 94, you muſt divide 10 into 2 Parts (to wit) 6 and 4, 6 being $\frac{1}{2}$, and 4 being $\frac{1}{3}$ of 12, and then ſay, the $\frac{1}{2}$ of 94 is 47, and the $\frac{1}{3}$ of 94 is 31 Feet 4 Inches, which being ſet down, you proceed to know the Product of 48 Feet multiplied by 11 Inches, which to do ſay thus, It is 48 Feet lacking 48 Inches, or 4 Feet, therefore I ſet down 44 Feet; behold the Example or Dimenſion caſt up in the Margin above.

Thus I have ſhewn you from 1 Inch to 12 Inches (beginning at 2 Inches, and ending at 11 Inches) how to work them into Feet, to ſave Multiplication and Diviſion; which way, after you are expert in it, will ſave you ſome Trouble and Time in caſting up many Dimenſions.

Some Things to be obſerved in meaſuring of Brickwork.

Sometimes Walls are wrought 2 Inches thicker than any of theſe Thickneſſes before ſpoken of.

Which 2 Inches ſerve for a Water-Table to the Wall, which is uſually ſet off, about 2 Feet above the Ground; therefore you may meaſure the Wall at the ſame Thickneſs that it is above the Water-Table, and add the 2 Inch-work to it thus:

Suppoſe a Wall 20 Feet in Length, and 2 B. thick above the Water-Table;

After you have taken the Dimenſion of the Wall from the Bottom to the Height that you are to take it, at 2 B. thick, then add 20 Feet in Length by the Height of the 2 Inch-work (to wit) from the Bottom to the ſetting off, or Water-Table, which being halfed is ſo much 4 Inch-work, and then reduce it to 1 $\frac{1}{2}$ B. work.

Secondly Note, That all kinds of ornamental Work in Brick, are generally done at a Rate by the Foot, except there be an Allowance of a Sum of Money over and above the Rate by the Rod, or except a good Rate is allowed by the Rod, and all the ornamental Work to be meaſured into the Rodwork.

By ornamental Work you are to underſtand ſtrait or circular Arches over Windows or Doors, Facio's with or without Moldings, Architrave round the Windows, or rubbed Returns, Friezes and Cornices, Ruſtick Quines, *&c.* In fine, all kind of Work that is hewed with an Ax, or rubbed upon a Stone, is ornamental Work, and ought to be paid for beſides the Rod-Work.

Thirdly Note, When you meaſure Arches, either ſtrait or circular, you muſt meaſure them in the Middle (that is to ſay) if a ſtrait Arch be 12 Inches in Height or Depth, you muſt meaſure the Length of it in the Middle of the 12 Inches, which Length will be longer than if you meaſure it on the under Side next the Head of the Window, by ſo much as one Side of the ſpringing of the Arch is skewed back from the Upright of the Jaum.

Alſo in circular Arches, obſerve that the upper Part of the Arch is more in Length (being girt about) than the under Part, becauſe it is a Segment of a greater Circle, cut off by the ſame right Line that the leſſer is, therefore it muſt be girt in the Middle.

Fourthly Note, That Facio's, Architrave Friezes, Cornices, Baſe Moldings and Plinths, are meaſured by the Foot, Line-meaſure (or Running-meaſure, as it is commonly called.)

Fifthly Note, You muſt take notice of the Jaums of Windows and Doors being ſplayed on the Inſide of Buildings, at the Rate by the Foot Running-meaſure.

Sixthly Note, Meaſuring of Tyling is the ſame as meaſuring of Roofing, which is taught at the beginning of this Tract in meaſuring of Carpenters Work; only there is this Difference, you muſt meaſure the Length of all the Vallies, and add ſo many Feet as they are in Length, to the Number of Feet in the Meaſurement of the Roof; you muſt alſo add the Eyes and Barge-courſes, Tyling being done by the Square like Roofing.

You may alſo meaſure Tyling by the Flat and half of the Building, after the ſame Manner that you are ſhewed to meaſure Roofing (in meaſuring of Carpenters Work) adding the Barge-courſes and Eves; alſo there you may ſee how to meaſure Hipt Roofs.

Seventhly Note, That Brick-paving is done by the Yard, and is brought into Yards by dividing the Product of the Feet by 9, as you have read in meaſuring of Painting, Plaiſtering, *&c.*

Eighthly Note, The meaſuring of Gable-ends in Brickwork, is the ſame as in the meaſuring of Carpenters Work, wherein you are taught to meaſure them.

Ninthly, Remember in the meaſuring of Walls joining to each other anglewiſe, that you take the Length of one Wall to the Outſide of the Angle, and the Length of the other to the Inſide of the Angle.

Tenthly, If you ſhould have a Gable-end, to meaſure, and you know the Width of the Houſe, or the Length of the Baſe Line of the Gable-end, and you deſire to know the Length of the Perpendicular, you may find it by the Proportions of the Table that is in the meaſuring of Carpenter's Work, and the Rule of Three.

Or briefer thus, Suppoſe the Width of the Gable-end to be 20 Feet, and you would know the Length of the Perpendicular; take the Length of the Rafter which is 15 Feet, being $\frac{3}{4}$ of the Width (for if the Roof be true Pitch, $\frac{3}{4}$ of the Breadth is the Length of the Rafter) to which add half the Length of the Rafter (*viz.*) 7 Feet and $\frac{1}{2}$, the Product is 22 Feet and $\frac{1}{2}$; the half whereof (to wit) 11 Feet and 3 Inches is the Perpendicular.

Although this is a Way commonly uſed, yet it is a ſmall Matter more than the Length of the Perpendicular, as by the former Rule will appear.

Thus much may ſuffice a diligent and practical Reader concerning Meaſuring, and of which you may ſee more in the *Appendix*.

As to the Meaſuring of Vaults and Arches, the following Treatiſes, intituled, *The Meaſuring of Plains* and *Bodies*, will inſtruct you, as alſo the *Appendix*.

And becauſe Brickwork is done at ſeveral Rates *per* Rod, and many times there happens a Number of Feet to remain, after you have divided your whole Product by 272.25 (to bring your Number of Feet into Rods) it will be convenient to ſhew how to find the Price of 1 Foot of Brickwork, or of any Number of Feet, at any Rate *per* Rod.

Having the Price per *Rod given, and the Number of Feet that you deſire to know what they come to, work thus:*

Firſt, Reduce the Price of a Rod into Shillings, by multiplying the Pounds by 20 (the Number of Shillings in one Pound) and if the Price of a Rod be Pounds and Shillings, then add the Shillings to the Product of the Pounds, being multiplied by 20, ſo you will have the Price of a Rod in Shillings: Next multiply the Number of Shillings by 12 (the Pence in one Shilling) and that brings the Price of a Rod into Pence. Then work according to the Rule of Three, ſaying,

Example 1.

If I have 5 *l.* for a Rod of Brickwork, what muſt I have for 34 Feet, according to that Rate?

State the Queſtion thus.

If a Rod, which is 272 Feet and $\frac{1}{4}$, or 272 $\frac{25}{100}$, give 5 *l.* what will 34 Feet give?

```
                              l.   s.
Multiply                      5    00
                        by         20
                             ---------
The Product is                    100 Shillings
Which multiply by                  12 Pence
                             ---------
And the Product is               1200 Pence
Which multiply by                  34 Feet
                             ---------
                                 4800
                                3600
                             ---------
And the Product is              40800
                             ---------
```

Which divide by 272.25, adding 4 Cyphers to the Product or Dividend.

```
         0 1 7
        2340
      268580
     135750000      d.
    4080000000  (149. 86/100
    2722555555
     2722222
      27222
       277
        2
```

And the Quotient is 149 *d.* and $\frac{86}{100}$ of a Penny, being more than 3 *q.* for the 34 Feet, at 5 *l. per* Rod.

Which 149 Pence divide by 12, and the Quotient is 12 *s.* and $\frac{5}{12}$, or 5 Pence 3 Farthings, the Rate of the 34 Feet deſired.

```
  2 (5 s.  d.  q.
149 (12    5   3
122
 1
```

Example 2.

If I have 5 *l.* 5 *s.* for a Rod, what comes 20 Feet to at that Rate?

State the Queſtion thus, and it ſaves you the Labour of multiplying by 20.

```
             F.          S.
If      272.25. give    105 what will 20 Feet give?
Multiply by              12 Pence
                        ———
                        210
                       105
                       ————
The Product is         1260 the Rate of a Rod in Pence,
which multiply by        20 the Number of Feet, And
                       ————
the Product is        25200 : To this Product add 4
                      —————
```

Cyphers, and divide by 272.25.

```
      1535
     69800
  252000000 (92  56
   27222555
    27222
     272
```

The Quotient is 92 *d.* and $\frac{56}{100}$ of a Penny, which 92 *d.* being divided by 12 *d.* gives you 7 *s.* 8 *d.* and the Fraction remaining namely $\frac{56}{100}$ is ſomewhat above 2 Farthings.

So at 5 *l.* 5 *s.* *per* Rod, 20 Feet comes to 7 *s.* 8 *d.* 2 *q.*

Example 3.

If I have 5 *l.* 5 *s.* for one Rod of Brickwork, what muſt I have for 1 Foot?

	l.
Multiply	05
By	20 Shillings
The Product is	100
Add the	5 Shillings
It is	105 Shillings
Which multiply by	12 Pence
	210
	105
The Price of a Rod is	1260 Pence

Which ſhould be multiplied by the Third Number in the Queſtion (to wit) 1 Foot; but becauſe multiplying any Number by 1 doth not encreaſe it,

Therefore divide 1260 *d.* by 272.25 Feet.

$$\begin{array}{l} \quad\;\; 220 \\ \quad\;\; \cancel{7}\cancel{6}\cancel{5}5 \\ \;\cancel{3}\cancel{7}\cancel{1}\cancel{0}\cancel{0}\cancel{0}0 \; d. \\ \cancel{1}\cancel{2}\cancel{6}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0} \\ \;\cancel{2}\cancel{7}\cancel{2}\cancel{2}\cancel{5}\cancel{5}\cancel{5} \\ \quad \cancel{2}\cancel{7}\cancel{2}\cancel{2}\cancel{2} \\ \quad\;\; \cancel{2}\cancel{7}\cancel{2} \end{array}$$

($\frac{62}{100}$ of a Penny, being more than 2 *q.* and not 3 *q.*

And the Quotient is 4 *d.* $\frac{62}{100}$ of a Penny.

Then to know the Value of the Fraction $\frac{62}{100}$, Multiply the Numerator 62 by 4 (the Number of Farthings in one Penny) the Product is 248, which divide by the Denominator 100, and the Quotient is 2 Farthings, and the Remainder $\frac{48}{100}$, of a Farthing; which Numerator 44, is leſs than half of the Denominator 100, and therefore leſs than half a Farthing.

Example.

The Numerator 62 of the Fraction
Multiply by 4 Farthings

The Product is 248, which divide
by the Denominator 100,

And the Quotient is 2 Farthings, and ſomething more. } 248 q. 100(2 $\frac{48}{100}$

Thus the Rate of 1 Foot of Brickwork after the Rate of 5 *l.* 5 *s.* *per* Rod, is found to be Fourpence half-penny half Farthing; which was required.

The ſame Rule holds in Tyling, or Carpenters Work, only changing the Number 272. 25. (the Feet in a Rod) and inſtead of it uſe 100 (the Number of Feet in a Square.)

The End of the FOURTH BOOK.

MEASURING

OF

Superficial PLAINS,

BY

Vulgar ARITHMETICK,
without reducing the Integers into the
leaſt Denomination.

TOGETHER WITH

Directions for Meaſuring

OF

LAND.

The FIFTH BOOK.

By *VEN. MANDEY.*

LONDON:
Printed in the YEAR M.DCC.XXVII.

MEASURING OF *Superficial* PLAINS.

BOOK V.

CHAP. I.

By meaſuring of Plains, is meant the finding the Area or Content of Superficial Figures, ſuch as are Quadrates, Triangles, Circles, &c. *and as a Line is made by the Motion of a Point, ſo likewiſe a Superficie is made by the Motion of a Line.*

IN meaſuring of Plains, the firſt Figure to begin with (as being moſt eaſy) is a Quadrate or Square, which is a Figure incloſed with four equal Sides, making four right Angles.

The next is an Oblong or long Square, which is a Figure having four right Angles, and four Sides, but not equal or of one Length. Two of which Sides being oppoſite, are of one Length and Parallel; and the other two Sides being of another Length, are oppoſite and parallel.

The meaſuring of theſe two kinds of Plains, you have been already ſhewn in the preceding Tract of Meaſuring (to wit, by multiplying the Length by the Breadth.)

PROP. I. *Fig.* I.

To meaſure (or find the Content of) a Rhombus, being a Figure like a Quarry of Glaſs, having four equal Sides; and two Pair of equal Angles.

SUppoſe the Rhombus ABCD in *Fig.* I. one Side whereof is 5 Feet, and 4 Inches; take this for your Length.

Then for the Breadth take the neareſt Diſtance between any two of the Sides, as ſuppoſe the Prick Line EF being 5 Feet, multiply theſe two, and the Product is 26 Feet, 8 Inches, which is the Area or Content of the Rhombus ABCD.

Otherwiſe thus,

Draw the Diagonal Line AC, which divides the Rhombus into two Triangles; next draw the Perpendicular Line DB.

Then take the Diagonal Line AC for your Length, and the half Length of the Perpendicular for your Breadth, and multiply them, and the Product is the Content.

See the Examples caſt up both Ways.

	Feet.	In.
The Length of one Side AB (the other three being each the ſame Length) is	5	4
The neareſt Diſtance (or EF) is	5	0
	25	
	1	8
Being caſt up the firſt Way, the Content is	26	8

The Second Way.

	F.	In.	P.
The Length of the Diagonal Line AC is	9	2	0
The half Length of the Perpendicular is	2	10	11
	18	4	
	7	6	
		1	8
		8	3
			1
The ſame Content as before	26	8	0

The multiplying of the Parts of Inches into the Feet and Inches, you are taught in the foregoing Tract of Meaſuring of Glaziers Work.

PROP.

PROP. II. *Fig.* II.

To meaſure (or find the Content of) a Trapezium, or four-ſided Figure, two ſides whereof are parallel.

SUppoſe the Trapezium ABCD in *Fig.* II. whoſe longeſt Side A B is 9 Feet, 4 Inches $\frac{1}{2}$; the Side parallel to it is DC, being 7 Feet, 2 Inches and $\frac{10}{12}$ of an Inch; the Breadth being taken the neareſt Way, is the prick Line whoſe Length is 6 Feet, and $\frac{1}{4}$ of an Inch.

Add the two parallel Sides together, whereof take half the Sum for your Length; then take the neareſt Diſtance between thoſe two Sides that are neareſt together for your Breadth, and multiply them, the Product whereof is the Content.

Example.

	F.	In.	P.	
The longeſt Side AB, is	9	4	6	
The ſhorteſt parallel Side DC, is	7	2	10	
Thoſe two being added, is	16	7	4	
The mean Length being one half, is	8	3	8	
This being mult. by the neareſt Diſtance	6	0	3	
	48			Sec.
	1	6		
		4		
		2		9
				2
The Content required is	50	0		11

The

See the Examples caſt up both Ways.

	Feet.	In.
The Length of one Side AB (the other three being each the ſame Length) is	5	4
The neareſt Diſtance (or EF) is	5	0
	25	
	1	8
Being caſt up the firſt Way, the Content is	26	8

The Second Way.

	F.	In.	P.
The Length of the Diagonal Line AC is	9	2	0
The half Length of the Perpendicular is	2	10	11
	18	4	
	7	6	
		1	8
		8	3
			1
The ſame Content as before	26	8	0

The multiplying of the Parts of Inches into the Feet and Inches, you are taught in the foregoing Tract of Meaſuring of Glaziers Work.

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Add the two parallel Sides together, whereof take half the Sum for your Length; then take the neareſt Diſtance between thoſe two Sides that are neareſt together for your Breadth, and multiply them, the Product whereof is the Content.

Example.

	F.	In.	P.	
The longeſt Side AB, is	9	4	6	
The ſhorteſt parallel Side DC, is	7	2	10	
Thoſe two being added, is	16	7	4	
The mean Length being one half, is	8	3	8	
This being mult. by the neareſt Diſtance	6	0	3	
	48			Sec.
	1	6		
		4		
		2		9
				2
The Content required is	50	0		11

The 11 beyond the Place of Parts of Inches is $\frac{11}{12}$ Parts of $\frac{1}{12}$ Part of an Inch, or $\frac{11}{144}$ Parts of an Inch, which is not conſiderable, except the Superficies that you meaſure be ſome rich or coſtly thing.

Note, That I account, or ſuppoſe the Inch to be divided into 12 Parts, for two Reaſons: The firſt is, The more Parts it is divided into, the nearer to the Truth you may take the Length and Breadth of any Superficies or Solid, and likewiſe proceed the nearer to the true Content.

The ſecond Reaſon is, Becauſe the Parts of Inches thus divided, have the ſame Proportion to the Inches, that the Inches have to the Feet, and by that means theſe Parts may be multiplied into the Feet and Inches; which according as the Inch is divided on our common Rulers into 8 Parts, thoſe Parts cannot be multiplied into the Feet and Inches, but the Length, and likewiſe the Breadth of any Superficies muſt be reduced into the leaſt Denomination, and then multiplied each by other, which cauſeth a great deal of Multiplication, and alſo of Diviſion, before you arrive at the Surperficial Area or Content, as you may ſee in the preceding Tract at the beginning of Meaſuring of Glaziers Work, from Page 216, to Page 219, where I have proved this Way of multiplying the Parts of Inches into the Feet and Inches to be true, both by Decimal Arithmetick, and alſo by reducing the Length and Breadth into the leaſt Denomination (to wit, Quarters of Inches.)

And indeed (as Mr. *Oughtred* ſaith in his Circles of Proportion) the dividing of the Inches on the Rulers, into Halfs, and Quarters, and Half-quarters, is moſt inartificial; and I ſuppoſe was done before Decimal Arithmetick was brought to light. Which if the Foot were divided into 10 Parts (which may as well be called Inches, as now it is divided into 12 Parts) and each of thoſe Parts into 10 Parts, and the Length and Breadth of Plains or Solids taken therewith, and ſet down Decimally, and caſt up by Decimal Arithmetick; the Science of Meaſuring would be much eaſier than now it is, and ſave a great deal of Labour.

Note, That a four-ſided Superficies which hath none of the Sides parallel, likewiſe every plain Figure of more Sides than four, being propoſed to be meaſured, muſt with Diagonal Lines be divided into Triangles.

And Note, That every ſuch Figure containeth ſo many Triangles as it hath Sides, abating two out of the Number (to wit, if the Figure to be divided into Triangles have ſix Sides, it will contain four Triangles, if ſeven Sides, five Triangles); then theſe Triangles are to be meaſured ſeverally, and their Contents being added, gives the Content of the irregular Figure.

PROP. III. *Fig.* III.

To meaſure any Triangle.

MUltiply the longeſt Side (being uſually called the Baſe) by half the Perpendicular (which is a Line let fall Perpendicular from the Angle, oppoſite to the Baſe) and the Product is the Content of the Triangle.

Suppoſe a Triangle ABC in *Fig.* III. whoſe Baſe AC is 7 Feet, the Side CB 6 Feet, and the Side BA 5 Feet, and the Perpendicular, which is the prick

Line 3 Feet, 2 Inches, the half whereof is 2 Feet, 1 Inch; which being multiplied by the Baſe Line 7 Feet, makes 14 Feet, 7 Inches, for the Content of that Triangle.

Example.

	F.	I.
The Length of the Baſe is	7	0
Half the Length of the Perpendicular is	2	1
The Content of the Triangle is	14	7

Otherwiſe thus:

Multiply the whole Perpendicular by half the Baſe.

Example.

	F.	I.
The whole Perpendicular is	4	2
Being multiplied by half the Baſe	3	6
	12	6
	2	1
The Content (as before) is	14	7

PROP. IV. *Fig.* IV.

The Side of an Equilateral Triangle being given, to find the Perpendicular Arithmetically.

LET the Side of the Triangle given be AC in *Fig.* IV. whoſe Length is 8 Feet.

Firſt ſquare the Side, then ſquare half the Baſe, and ſubtract that from the Square of the Side, and the ſquare Root of the Remainder, is the Length of the Perpendicular.

Example.

	F.	I.
The given Side is A C	8	o
Which to ſquare, mnltiply by	8	o
The Square of the Side A C is	64	o
The whole Baſe is 8, the half whereof is	4	o
Which to ſquare, multiply by	4	o
The Square of half the Baſe is	16	o
Which ſubtract from	64	o
The Square of the given Side	16	o
and the Remainder is	48	o

The Square Root whereof is 6 Feet, 11 Inches, and $\frac{2}{12}$ Parts of an Inch *Fere*, being the Length of the Perpendicular required.

It is ſuppoſed that the Reader hereof can extract Square and Cube Roots; if he cannot, he may learn it in Mr. *Wingate*'s Arithmetick, in which Book they are plainly and briefly demonſtrated; or he may work by Logarithms.

PROP.

PROP. V. *Fig.* IV.

The Perpendicular and Side of an Equilateral Triangle being given, to find the Content of that Triangle.

IN the former Diagram, *Fig.* IV. the Perpendicular 6 Feet, 11 Inches $\frac{2}{12}$ *Fere*, and the Side 8 Feet being given, the Content is required.

Multiply the whole of either, by half the other, and the Product is the Content required.

Example.

	F.	I.	P.	
The whole Baſe or Side is	8	0	0	
The half of the Perpendicular is	3	5	7	*Fere*
	24			
	3	4		
		4	8	
The Content of the Triangle is	27	8	8	*Fere*

If you multiply the whole Perpendicular 6 Feet, 11 Inches, by half the Side 4 Feet, it produceth the ſame as before (to wit) 27 Feet, 8 Inches, $\frac{8}{12}$ of an Inch almoſt, for *Fere* ſignifies almoſt or near.

To find the Content of the ſame Equilateral Triangle, without knowing the Length of the Perpendicular.

Make it as 30 to the Square of one Side, ſo 13 to a fourth Number, which will be the *Area* requir'd.

Example.

64	$\cancel{2}\cancel{2}$	F.
13	$\cancel{8}\cancel{3}\cancel{2}$ (	27 $\frac{2}{30}$ the ſame as before.
192	$\cancel{3}\cancel{3}0$	
64		
832		

PROP.

PROP. VI. *Fig.* V.

The Perpendicular and Baſe of a right angled Triangle being given, the Content of that Triangle is required.

IN *Fig.* V. the Perpendicular FG, being 12 Feet, and the Baſe CE 25 Feet, is given, and the Content required.

Multiply the whole of either by half the other, as 12 Feet by 12 Feet, 6 Inches, or 25 Feet by 6, the Product is 150 Feet, the Superficial Content required.

Or multiply the whole by the whole, the Product is 300, whereof take half, it is 150 as before.

Or thus, without the Perpendicular.

Multiply the two Sides 15 by 20, the Product is 300, whereof take half for your Demand; or multiply half of one Side by the whole of the other Side, the Product is 150, as before.

Theſe Rules hold good in thoſe right-angled Triangles, whoſe longeſt Side (or Baſe) being ſquared, the Product is as much as the Square of both the other Sides, as in *Fig.* V. the Square of CE is 625, the Square of CF is 225, and the Square of FE is 400; which two laſt Squares being added, make 625, the Square of the Baſe; ſo likewiſe, in a right-angled Triangle, whoſe Sides are 6 Feet, 8 Feet, and 10 Feet, it holds the ſame Proportion; for the Square of 10, which is 100, is as much as the Square of 6, which is 36, and the Square of 8, which is 64, being added together.

PROP. VII. *Fig.* VI.

The Perpendicular and Baſe of an Iſoſceles Triangle given, to find the Area or Superficial Content.

IN *Fig.* VI. add all the three Sides together, whereof take half, then take the Difference of each Side from that half, and multipiy thoſe three Differences one by another, the Product whereof multiply by the half Sum of the three Sides, and the Square Root of that Product is the Superficial Content.

Example.

The 3 Sides being added, make 56, the half whereof is 28, then the Difference of the Side A B (being 20) from 28 is 8; the like Difference is between the Side B C and 28, namely 8; and the Difference between the Side AC (being 16) and 28, is 12. Now theſe 3 Differences being multiplied one by another, namely, 8 by 8 is 64, and 64 by 12, makes 768, which being multiplied by 28, the half Sum of the 3 Sides, the Product is 21504, whereof the Square Root is 146 Feet, 7 Inches, 8 Parts, and $\frac{9}{12}$ of a Part, or 146 Feet, 7 Inches, and $\frac{72}{144}$ of an Inch.

Which Rule is general for all right-lined Triangles whatſoever.

PROP. VIII. *Fig.* VII.

To find the Perpendicular of any Triangle Arithmetically, the Sides being given.

IN *Fig.* VII. let the Sides given be ED 6 Feet, DF 8 Feet, and FE 10 Feet, it is required to find the Perpendicular?

Square the 3 Sides ſeverally, then add the Square of the Baſe (or longeſt Side) to the Square of one of the other two Sides, as to the Square of ED, from whence ſubtract the Square of the remaining Side, and from the Remainder take half, which divide by the Baſe, the Quotient thereof being ſquared, deduct or ſubtract it from the Square of the ſhorteſt Side, and the Square Root of the Remainder is the Length of the Perpendicular.

Example.

The Square of ED is 36, the Square of DF is 64, and the Square of FE is 100; then the Square of the Baſe 100 being added to the Square of ED 36, makes 136, from whence the Square of DF being ſubtracted, which is 64, the Remainder is 72, whereof the half is 36, which being divided by the Baſe 10, produceth 3 Feet $\frac{3}{5}$; or 3 Feet, 7 Inches, 2 Parts, and $\frac{4}{10}$ of a Part, for the leſſer Segment of the Baſe EG; the Square of which Segment is 12 Feet $\frac{24}{25}$, or 12 Feet, 11 Inches and 6 Parts, and being ſubtracted from the Square of ED 36, the Remainder is 23 Feet $\frac{1}{25}$, or 23 Feet and $\frac{1}{12}$ of an Inch +, whoſe Square Root is 4 Feet, 9 Inches, and 7 Parts +, is the Length of the Perpendicular DG required. But this being very troubleſome, I will give you a Table in the *Appendix*, which will ſhew the Length of ſeveral Perpendiculars.

PROP. IX. *Fig.* VIII.

The Side of a Pentagon being given, to find the Area or Content.

IN *Fig.* VIII. let the given Side be 10, it is required to find the Area of that Pentagon.

Deſcribe an Iſoſceles Triangle on any Side of the Pentagon, whoſe Top is the Centre of the Pentagon, as ADB, whoſe Perpendicular being found is 6 Feet, 10 Inches, and 5 Parts + of an Inch, divided into 12 Parts, which being multiplied in half the Perimetry 25, produces 171 Feet, 8 Inches, and 5 Parts, for the Area required.

This Rule is general in all kind of regular Polygons, of how many Sides ſoever, as well for their Superficial Content, as finding their Perpendicular.

But becauſe it is troubleſome finding the Centre Geometrically, I will here ſhew how to find it by Arithmetick.

The Side of a Pentagon being given, to find the Radius of a Circle inſcribed within that Pentagon.

The Rule is this, As 182 to 125, ſo is the Side of the Pentagon, to the Semidiameter of a Circle inſcribed in it.

Example.

The Side given is 10 Feet, what is the neareſt Diſtance from the Centre of the Pentagon to one of the Sides?

Multiply and Divide according to the Rule of Three, and you will find 6 Feet, 10 Inches, and 5 Parts +.

PROP. X. *Fig.* IX.

The Diameter of a Circle being given, to find the Circumference thereof Arithmetically.

THE Proportion of the Diameter to the Circumference was firſt found out by *Archimedes*, who brought the Length of the Perimeter of a Circle within the Limits of Numbers, very little differing from the Truth, demonſtrating the ſame to be leſs than three Diameters and a ſeventh Part, but greater than three Diameters and $\frac{10}{71}$ Parts of the Diameter, ſo that ſuppoſing the Radius to conſiſt of 10000000 equal Parts, the Arch of a Quadrant will be between 15714285 and 15704225 of the ſame Parts. But ſince *Ludovicus Van Cullen* hath come yet nearer to the Truth, and pronounced from true Principles that the Arch of a Quadrant (putting as before 10000000 for the Radius) differs not one whole Unite from the Numb. 15707963. But for our purpoſe we ſhall make choice either of the Proportion in whole Numbers by *Archimedes*, which is as 7 to 22, or of the Proportion which *Metius* hath given, which is as 113 to 355, ſo is the Diameter to the Periphery or Circumference.

Let the Diameter given be 14 Feet, in *Fig.* IX. it is required to find the Circumference thereof.

Multiply the Diameter given (being 14) by 22, the Product is 308, which divided by 7, bringeth 44 Feet for the Circumference required; or multiply the Diameter by $3\frac{1}{7}$ the Product is 44, as before; but this makes the Circumference a ſmall Matter too great, as you may ſee if you multiply 14 by 355, the Product is 4970, which being divided by 113, the Quotient is 43 and $\frac{111}{113}$ the Remainder, which is 111 Parts of an Unite (whether it be a Foot, or an Inch, or any other Meaſure) divided into 113 Parts.

If the Circumference be given, and the Diameter required, it appeareth by this Rule, that the Circumference 44 being multipily'd by 7, and the Product divided by 22, brings 14 the Diameter.

PROP. XI. *Fig.* IX.

The Diameter and Circumference of a Circle being given, to find the Area or Superficial Content thereof Arithmetically divers ways.

LET the Diameter of the Circle be 14, as in *Fig.* IX. and the Circumfererence thereof 44. It is required to find the Superficial Content.

Multiply half the Diameter by half the Circumference, it ſhews the Content of any Circle.

Thus the half Circumference is 22, which being multiplied by the Semidiameter 7, the Product is 154, the Superficial Content required.

Or multiply the whole Circumference 44, by the Semidiameter 7, the Product will be 308, whereof take half, which is 154, as before.

Or multiply the Square of the Diameter 196 by 11, the Product will be 2156; which divide by 14, brings 154, as before.

PROP. XII. *Fig.* IX.

Having only the Diameter of a Circle given, to find the Content.

SAY, as 7 is to 22, ſo is the Square of the Semidiameter to the Content of the Circle.

Let the Diameter given be 14, as in *Fig.* IX.

Then ſay by the Rule of Three, if 7 give 22, what will 49 give? (which 49 is the Square of the Semidiameter;) multiply the third Number 49, by the ſecond Number 22, the Product is 1078, which divide by the firſt Number 7, and you have 154 for the Content, as before.

PROP. XIII. *Fig.* IX.

Having the Circumference given, to find the Content.

THE Rule is at 88 (which is 4 times 22) is to 7, ſo is the Square of the Circumference to the Content.

Example.

Let the Circumference be 44, the Square thereof is 1936; then ſay by the Rule of Three,

If 88 give 7, what will 1936 give?

Multiply 1936 by 7

The Product is 13552

Which divide by the firſt Number 88, and the Quotient is 154 Feet.

$$\begin{array}{cccccl}
 & \cancel{4} & \cancel{3} & & & \\
 & \cancel{5} & \cancel{7} & \cancel{3} & & \\
\cancel{1} & \cancel{3} & \cancel{5} & \cancel{5} & \cancel{2} & (154 \text{ Feet, the Content of the Circle.}\\
 & \cancel{8} & \cancel{8} & \cancel{8} & \cancel{8} & \\
 & & \cancel{8} & \cancel{8} & &
\end{array}$$

PROP. XIV. *Fig.* IX.

The Content of a Circle being given, to find the Diameter.

THE Rule is as 22 to (4 times 7, which is) 28, ſo is the Content to the Square of the Diameter.

Example.

Let the Content given be 154, and it is required to find the Diameter.

If 22 give 28, what will 154 give? Multiply and Divide, and you will find 196, the Square Root whereof is 14, the Diameter required.

PROP. XV. *Fig.* IX.

The Content of a Circle being given, to find the Circumference.

THE Rule is, as 7 is to (4 times 22, which is) 88, ſo is the Content to the Square of the Circumference.

Example.

Let the Content given be 154, and the Circumference required to be found.

Then ſay, by the Rule of Three,

If 7 give 88, what will 154 give? Multiply and divide, and you will find 1936, the Root whereof being 44, is the Circumference required.

PROP. XVI. *Fig.* IX.

The Content of a Circle being given, to find the Side of a Square, the Content of which Square ſhall be equal to the Content of the Circle.

EXtract the Square Root of the Content given, and that Root is the Side of the Square required.

Example.

Let the Content given be 154, the Square Root whereof is 12 Feet, 4 Inches $\frac{11}{12}$ Parts of an Inch, which is the Side of a Square, whoſe Content is equal to the Content of the Circle given.

PROP. XVII.

The Diameter of any Circle being given, to find the Side of a Square, the Area of which Square ſhall be equal to the Area of the Circle of the given Diameter.

THIS Propoſition may be ſolved ſeveral Ways, as firſt according to *Archimedes* his Proportion in whole Numbers, thus:

As 7 is to 22, ſo is the Square of the Semidiameter to the Content required.

Let the Diameter given be 7 Feet, the Semidiameter will be 3 Feet, 6 Inches, the Square whereof is 12 Feet, 3 Inches; which being multiplied by 22 Feet, the Product is 269 Feet, and 6 Inches, which divided by 7, makes 38 Feet, 6 Inches for the Content of the Circle, which Content is a ſmall Matter more than the true Content.

Secondly, According to *Metius* his Proportion, thus:

As 113 to 355, ſo is the Square of the Semidiameter to the Content required.

Example.

The Square of the Semidiameter aforeſaid is 12 Feet, 3 Inches, which being multiplied by 355 Feet, produces 4348 Feet, 9 Inches; which Product being divided by 113 Feet, makes 38 Feet, 5 Inches, 9 Parts, and $\frac{9}{12}$ of a Part, which Content is leſs than the Content produced from *Archimedes's* Proportion, by more than $\frac{2}{144}$ Parts of a Superficial Foot, and the greater the Diameter is, the more the Difference will be.

Or thus, As 452 (which is 4 times 113) is to 355: So is the Square of the Diameter to the Content required.

Thirdly, It may be ſolved by Decimal Arithmetick, thus; as 1 Foot to Feet 3.1416, ſo is the Square of the Semidiameter to the Content required.

Example.

Feet 3.1416 being multiplied by Feet 12.25, produceth Feet 38.484600, which ſhould be divided by the firſt Number 1; but becauſe 1 doth neither increaſe any Number being multiplied by it, nor diminiſh any Number being divided by it, therefore we find the Content to be Feet 38.484600, which being reduced to Feet, Inches, and Parts, is 38 Feet, 5 Inches, 9 Parts, 9 Seconds, and 4 Thirds.

Fourthly, It may be ſolved Decimally thus:

As Feet 114.5915492 is to 360 Feet, ſo is the Square of the Semidiameter to the Content required.

Example.

360 Feet being multiplied by Feet 12.25, produceth Feet 4410.00 which being divided by Feet 114.5915492, produceth Feet 38.48451, which being reduced, is 38 Feet, 5 Inches, 9 Parts, 9 Seconds, and 2 Thirds; which is a ſmall Matter leſs than the former Content.

But

But peradventure the Reader hereof may not underſtand Decimal Arithmetick, I will therefore exhibit one Way more by Vulgar Arithmetick, for the ſolving of this Propoſition ; which is this :

Multiply the Diameter given, by 10 Feet, 7 Inches, 7 Parts, 5 Seconds ; and divide the Product by 12 Feet, it produces the Side of a Square, which being multiplied by itſelf, is the Content of the Circle required.

Which I look upon to be as quick a Way as by Decimals, eſpecially if the Diameter given conſiſts of Feet, Inches, Halfs, Quarters, &c. and the Solution be required in the ſame, and cometh within a Trifle as near the Truth.

And here I ſhall a little inſiſt upon the multiplying the Fractional Parts of Inches into one another, for two Reaſons.

Firſt, As being a Way very uſeful, and ſaves a great deal of Labour (to wit) of rediciing the Integers into the leaſt Denomination (or Fractional Parts) it being the uſual Way ſo to do, among thoſe who underſtand not Decimal Arithmetick.

Secondly, Becauſe hitherto I have had occaſion only to multiply Twelfths of Inches into Feet and Inches ; and having occaſion now to multiply other Twelfths into thoſe Twelfths, and other Twelfths into theſe Twelfths, &c. which I name thus,

Feet, *Inches*, *Parts*, *Seconds*, *Thirds*, *Fourths*, &c.

And here Note, that as a Foot contains 12 Inches Line meaſure, ſo an Inch is ſuppoſed (or accompted) to contain 12 Parts Line meaſure (although upon the Rulers the Inches are divided but into 8 Parts, or Half-Quarters)

Quarters) and each Part to contain 12 Seconds, and each Second to contain 12 Thirds, and each Third to contain 12 Fourths, *&c.* And ſo the Proportion might be continued, if there were occaſion, to Fifths, Sixths, *&c.*

But according to ſuperficial Meaſure, a Foot contains 144 Inches, and an Inch 144 Parts, and a Part 144 Seconds, *&c.*

Alſo obſerve, that each Place of Twelfths is diſtinguiſhed by having a Letter put over it; thus, over the Feet write F, over Inches I, over Parts P, over Seconds S, over Thirds T, over Fourths Fo, *&c.* as beneath.

And for the better underſtanding how to multiply the Fractions or Twelfths into one another, obſerve the Rules in the following Table.

	F.	I.	P.	S.	T.	F.
1. *Note*, That Inches mult. into F. every 12 of the Product is and any Number leſs than 12 is Inches	1					
2. In. nnult. into (or by) In. every 12 of the Prod. is and any Number of the Prod. leſs than 12 are Parts	0	1				
3. Parts mult. into Feet, every 12 of the Prod. is and any Number under 12 are Parts.	0	1				
4. Parts multiplied into Inches, every 12 of the Prod. is and any Number of the Prod. under 12 are Seconds	0	0	1			
5. Parts multiplied by Parts, every 12 of the Prod. is and any Number leſs than 12 are Thirds	0	0	0	1		
6. Seconds multiplied into Fe. every 12 of the Prod. is and any Number leſs than 12 are Seconds	0	0	1			
7. Sec. mult. into Inches, every 12 of the Prod. is and any Number leſs than 12 are Thirds.	0	0	0	1		
8. Sec. mult. into Parts, every 12 of the Prod. is and any Number leſs than 12 are Fourths.	0	0	0	0	1	
9. Sec. multiplied into Sec. every 12 of the Prod. is and any Number leſs than 12 are Fifchs.	0	0	0	0	0	1
10. Thirds multiplied into Feet, every 12 of the Prod. is and any Number leſs than 12 are Thirds.	0	0	0	1		
11. Thirds multiplied into Inches, every 12 of the Prod. is and any Number leſs than 12 are Fourths.	0	0	0	0	1	
12. Thirds multiplied into Parts, every 12 of the Prod. is and any Number leſs than 12 are Fifths.	0	0	0	0	0	1
13. Fourths multiplied into Feet, every 12 of the Prod. is and any Number leſs than 12 are Fourths.	0	0	0	0	1	

Farther in Fractional Parts you need not proceed.

Here *Note*, if the greateſt Denomination of the Dimenſion to be caſt up be Inches, and the Content be required in ſuperficial Inches; then Parts multiplied into Inches, every 12 of the Product is one Inch; and Seconds multiplied into Parts, every 12 is one Part, *&c.*

Explanation of the Table in 3 *or* 4 *Examples.*

Firſt, Suppoſe you have 6 Parts to be multiplied into 7 Feet;

Say 6 times 7 is 42; then ask yourſelf how many times 12 is 42? and the Anſwer is 3 times, and 6 remaining; therefore according to the 3d Rule of the preceding Table, ſet 3 in the Place of Inches, noted with the Letter I, and the 6 that remains, being leſs than 12, you muſt write in the Place of Parts, noted above the Dimenſion with the Letter P.

Secondly, Suppoſe 8 Thirds to be multiplied into 8 Inches, ſay 8 times 8 is 64, which contains 5 twelves, and 4 remaining; therefore, according to the 11th Rule, ſet 5 in the Place of Thirds, noted by T; and the 4 that remains, ſet in the Place or Column of Fourths, noted with Fo.

Thirdly, Suppoſe you have 10 Seconds to be multiplied by 11 Parts, ſay 10 times 11 is 110, which contains 9 times 12, and 2 remaining; then, according to the 8th Rule of the foregoing Table, ſet down 9 in the Place of Thirds, noted on the Top with T, and the 2 that remains ſet in the Place of Fourths, noted by Fo, *&c.*

Theſe being premiſed, I come now to ſolve the foregoing *Prop.* (to wit) the Diameter of a Circle being given, whoſe Length is 7 Feet, it is required to find a Square, whoſe Content ſhall be equal to that Circle.

The Proportion is, as 12 Feet is to 10 Feet 7 Inches, 7 Parts, 5 Seconds; ſo is the Diameter to the Side of a Square, or the Square of the Diameter to the Content required.

The Rule thus; Multiply the Diameter by 10 Feet, 7 Inches, 7 Parts, 5 Seconds, and divide the Product by 12 Feet. The Quotient, together with the Remains, if there happen to be any, is the Side of a Square, which being multiplied in itſelf, is equal to the Content required.

Example.

	F.	I.	P.	S.	T.	Fo.
The Length of the Diam. given is	7	0	0	0	0	0
Which multiply by	10	7	7	5		
The Prod. of 10 F. by 7 F. ss	70					
The Prod. of 7 Inches by 7 F. is	4	1				
The Prod. of 7 Parts by 7 F. is	0	4	1			
The Prod. of 5 Sec. by 7 F. is	0	0	2	11		
The Total Product is	74	5	3	11		

Which Product, according to the foregoing Rule, muſt be divided by 12. Therefore divide the 74 Feet, by 12 Feet, thus,

Inches

~~1~~ (2 Inches.
~~7~~ ~~4~~(6 Feet.
~~1~~ ~~2~~

And the Quotient is 6 Feet, and 2 remains, which is 2 Inches, ſo the Side of the Square equal to the Circle, is 6 Feet, 2 Inches, 5 Parts, 3 Seconds, 11 Thirds; for inſtead of dividing the 5 Inches, 3 Parts, 11 Seconds (that belonged to the 74 Feet in the Total Product) by 12, only do thus; Remove them one Place forwarder towards the Right Hand, it is the ſame as if you had divided them by 12; ſo then the 5 Inches will ſtand in the Place of Parts, and be 5 Parts, and the 3 Parts will be 3 Seconds, and the 11 Seconds will be 11 Thirds; which obſerve as a general Rule in any Number to be divided after this manner.

When you have divided the Feet by 12, if there happen to be no Inches remaining, then ſet down your Feet under the Place of Feet, and put a Cypher in the Place of Inches, and then ſet your other Fractions next; as in this laſt Example, if there had been no Remainder beſides the Quotient, then you muſt have put 0 in the Place of Inches, and then the 5 Inches which belonged to the 74 Feet will be changed into 5 Parts, as if they had been divided by 12. And likewiſe the 3 Parts will be turned into 3 Seconds, &c.

	F.	I.	P.	S.	T.	Fo.
So that 74 Feet, 5 Inch. 3 Parts, 11 Seconds, being divided by 12, is	6	2	5	3	11	
Which is the Side of a Square; which multiply by	6	2	5	3	11	
The Prod. of 6 Feet by 6 Feet, is	36					
The Prod. of 2 Inch. by 6 Feet, is	1					
The ſame again is	1					
The Prod. of 2 Inch. by 2 Inch. is	0		4			
The Prod. of 5 Parts by 6 Feet, is		2	6			
The ſame again is		2	6			
The Prod. of 5 Parts by 2 Inch. is	0	0	0	10		
The ſame again is				10		
The Prod. of 5 Parts by 5 Parts, is				2	1	
The Prod. of 3 Sec by 6 Feet, is			1	6		
The ſame again is			1	6		
The Prod. of 3 Sec. by 2 Inch. is					6	
The ſame again is					6	
The Prod. of 3 Sec. by 5 Parts, is					1	3
The ſame again is					1	3
The Prod. of 11 Thirds by 6 F. is				5	6	
The ſame again is				5	6	
The Prod. of 11 Thirds by 2 In. is					1	10
The ſame again is					1	10
The Total Product is	38	5	9	10	7	2

Farther you need not proceed, the Value of the Products being inconſiderate.

Which is the Content required, and agrees with the Decimal way within $\frac{1}{1728}$ Parts of an Inch.

Although I have ſet the Product of each Multiplication one under another, for Demonſtration; yet nevertheleſs when you are perfect in caſting up, you may ſet the Products of 4 or 5 Multiplications in one level Line.

Let a ſecond Example be,

A Diameter given, whoſe Length is 42 Feet, and the Content of that Circle is required.

According to *Archimedes* his Proportion in whole Numbers, working as is ſhewn before, the Content will be 1386 Feet.

And according to *Metius*, in whole Numbers the Content will be found to be 1385 Feet and $\frac{50}{113}$ of a Foot, which reduced, is 1385 Feet, 5 Inches, 3 Parts, 8 Seconds.

Let us try what the Content will be our Way.

	F.	I.	P.	S.	T.	F.
The Diameter is	42	0	0	0	0	0
Which multiply by	10	7	7	5		
The Prod. of 10 F. by 42 F. is	420					
The Prod. of 7 Inch. by 42 Feet, is	24	6				
The Prod. of 7 Parts by 42 Feet, is	0	24	6			
The Prod. of 5 Sec. by 42 Feet, is			17	6		
The Total Product is	446	7	11	6		

Which 446 Feet, divide by 12.

~~4~~
~~1~~8(2 Inches.
~~446~~(37 Feet.
~~122~~
~~1~~

The Quotient is 37 Feet and 2 Inches, to which add the 7 Parts 11 Seconds 6 Thirds, by changing their Places one Degree forwarder toward the Right Hand, (inſtead of dividing them by 12) and it will be

	F.	I	P.	S.	T.	Fo.
For the Side of the Square	37	2	7	11	6	0
Which to ſquare, multiply by	37	2	7	11	6	0
The Prod. of 7 Feet by 37 is	259					
The Prod. of 30 F. by 37 F. is	111					
The Prod. of 2 Inch. by 37 Feet is	6	2				
The ſame again is	6	2				
The Prod. of 2 Inch. by 2 Inch. is			4			
The Prod. of 7 Parts by 37 Feet, is		21	7			
The ſame again is		21	7			
The Prod. of 7 Parts by 2 Inches, is			1	2		
The ſame again is			1	2		
The Prod. of 7 Parts by 7 Parts, is				4	1	
The Prod. of 11 Sec. by 37 Feet, is			33	11		
The ſame again is			33	11		
The Prod. of 11 Sec. by 2 Inch. is				1	10	
The ſame again is				1	10	
The Prod. of 11 Sec. by 7 Parts, is					6	5
The ſame again is					6	5
The Prod. of 11 Sec. by 11 Sec. is						10
The Prod. of 6 Thir. by 37 Feet, is				18	6	
The ſame again is				18	6	
The Prod. of 6 Thir. by 2 Inches, is					1	
The ſame again is					1	
The Prod. of 6 Thir. by 7 Parts, is						3
The ſame again is						3
The Content or Total Prod. is	1385	5	8	0	1	2

Which Content is four Parts more than the Content, according to *Metius's* Proportion, but near four Parts leſs than the Content according to *Archimedes's* Proportion.

And by the way, note, That a Circle of 14 Feet Diameter, contains 4 times as much as a Circle that is 7 F. Diameter. The Rule is thus, the Diameter 7 is contained in the Diameter 14 two times; now the Square of 2 is 4, therefore a Circle whoſe Diameter is 14, contains a Circle whoſe Diameter is 7 four times.

Likewiſe the Diameter in the laſt Example being 42, contains the Diameter 7 ſix times; for the Square of 6 is 36, therefore a Circle whoſe Diameter is 42, contains 36 Circles of 7 Feet Diameter apiece.

For if you multiply 38 Feet, 5 Inches, 9 Parts, 10 Seconds, + (being the Content of a Circle whoſe Diameter is 7) by 36, you will find it produce the Content of a Circle whoſe Diameter is 42.

Example.

	F.	I.	P.	S.	T.
A Circle whoſe Dia. is 7 F. the Con. is	38	5	9	10	6
Which multiply by	36				
	228	0	0	0	0
	114				
	15	27	30	18	0
A Cir. whoſe Dia. is 42 F. contains	1385	5	7	6	0

Let the Third Example be

A Diameter whoſe Length is 38 Feet, 6 Inches, and an half and half a quarter, and it is required to find the Content of that Circle.

The half Inch is 6 Parts, but the half quarter is ⅛ Part of an Inch, which muſt be reduced to Twelfths, thus;

1

thus; Multiply the Numerator of the Fraction (to be reduced) by 12, and divide the Product by the Denominator of the ſame, and the Quotient is the Numerator of a Fraction that hath 12 for his Denominator; and if any Number remain beſides the Quotient, multiply that Remainder by 12, and divide the Product by 8, ſo long until you have no Remainder beſides the Quotient.

Example.

$\frac{1}{8}$ of an Inch is to be reduced; multiply the Numerator 1 by your new Denominator 12, it is ſtill 12; divide it by the Denominator 8, the Quotient is 1, and 4 remains; the 1 in the Quotient is 1 Part, which add to the $\frac{1}{2}$ Inch or 6 Parts, and then it makes 7 Parts: Then multiply the Remainder 4 by 12, the Product is 48; which divide by the Denominator 8, and the Quotient is 6 and no Remainder, which is 6 Seconds. So $\frac{1}{8}$ of an Inch being reduced to 12ths, is 1 Par. 6 Sec.

	F.	I.	P.	S.	T.
Now I ſet down the Diameter thus	38	6	7	6	0
Which I multiply by	10	7	7	5	
	380				
	5	3	6		
	22	2	4	1	
		5	10		
		22	2	4	
			3	6	
			5		
			15	10	
				3	6
				2	6
The Product is	409	11	11	3	0

Which 409 divide by 12.

	F.	I.	P.	S.	T.	
~~1~~ ~~4~~(1 In. The Side of the Sq. is	34	1	11	11	3	
~~409~~(34 F. Which multiply by	34	1	11	11	3	
~~122~~						
~~1~~	136	31	2			
	1022	10	1			
	2	10		11	10	
		31	2	11	10	
				10		
			31	2	11	
			31	2	11	10
				8	6	
				8	6	
The Content required is	1167	3	11	8	6	10

This laſt *Example* could not be ſolved ſo near the Truth by vulgar *Arithmetick* any other way, without reducing the 38 Feet, 6 Inches, and half Inch into half quarters of Inches, which is a long way about.

PROP. XVIII.

The Superficial Content of a Circle being given; to find the Diameter.

LET the Content given (being found according to *Archimedes*'s Proportion) be 346 Feet 6 Inches; and the Diameter required.

The Proportion is as 11 is to 14, ſo is the Content given, to the Square of the Diameter required.

The Rule thus, Multiply the Content given by 14, the Product whereof divide by 11, and the Square Root of the Quotient is the Length of the Diameter.

Example.

Multiply the Content given, being 346 Feet 6 Inches, by 14, the Product is 4851, which being divided by 11, the Quotient is 441 for the Square of the Diameter, the Roof whereof is 21, the Length of the Diameter requir'd.

Or thus, and more exact, As 355 to 452, ſo is the Content given, to the Square of the Diameteter.

Example.

Multiply the Content given, being 346 Feet 6 Inches, by 452, the Product is 156618, which being divided by 355 produces 441 Feet and $\frac{63}{355}$ of a Foot, which, when reduced, is 441 Feet 2 Inches 1 Part, 6 Seconds, the Square Root whereof is 21 Feet 1 Inch 5 Parts 6 Seconds, the Diameter required.

To thoſe who underſtand Decimals, the Proportion for ſolving the 18th Propoſition is this,

As 1.00000 is to 1.27324, ſo is the Content given to the Square of the Diameter.

Therefore if you multiply 1.27324 by the given Content, the Square Root of that Product will be the Diameter required.

PROP. XIX.

The Diametor of a Circle being given; to find the Side of a Square which may be inſcrib'd within that Circle.

THE Rule is this: Square the Diameter, then take one half of the Product, then find the Root of that Product, it will be the Side of the Square deſired.

Example.

Let the Diameter given be 28 Feet 3 Inches, the Square whereof is 798 Feet 9 Parts, the Half whereof is 399 Feet 4 Parts and 6 Seconds; the Square Root whereof is 19 Feet 11 Inches 8 Parts 5 Seconds +, for the Length of the Side of the Square required.

Decimally thus, As 1.000000 is to .707106, ſo is the Diameter to the Side required.

Therefore if you multiply the ſaid .707106 by the Diameter given, the Product will be the Side of the inſcrib'd Square requir'd; for, as you read before, if you ſhould divide the Product by 1.000000, it will ſtill be the ſame.

PROP. XX. *Fig. X.*

The Diameter and Arch-Line of a Semi-circle being given; to find the Area thereof.

LET ABC in *Fig. X.* be a Semi-circle given, whoſe Diameter is AC, and the Arch-Line ABC, it is required to find the Area of that Semi-circle.

Multiply half the Arch-Line 11 by the Semidiameter 7, the Product will be 77 for the Area required.

PROP. XXI. *Fig. X.*

The Semidiameter and Arch-Line of a Sector of a Circle being given; to find the Area.

IN *Fig.* X. let BCD be the Sector of a Circle, whoſe Semidiameter is DC, or DB, and the Arch-Line BC, and it is required to find the Area.

Multiply the Semidiameter 7, by half the Arch-Line BC 5 Feet 6 Inches, the Product is 38 Feet 6 Inches for the Area required.

PROP. XXII. *Fig. X.*

Any Segment or part of a Circle being given, to find the Content.

IN *Fig. X.* let AF BE be the Segment of a Circle, the Content whereof is required.

Firſt find out the Centre of the Arch-Line AFB, (by the 21ſt Prop. of the 20th Page of Geometry) then draw the Lines DA and DB, and caſt up the whole Figure AFBD, as in the laſt Prop. which will be 38 Feet 6 Inches, then find the ſuperficial Content of the Triangle ABD, which is 24 Feet 5 Inches 9 Parts, and deduct the ſame out of the whole Content 38 Feet 6 Inches; the Remains is 14 Feet and 3 Parts, for the Content of the given Segment, which was required.

By this Rule (obſerved with Diſcretion) may all manner of Segments or Parts of a Circle, whether greater or leſs than a Semi-circle, be eaſily meaſured.

PROP. XXIII. *Fig.* X.

To find the Diameter of a Circle, by having one Part of the Diameter given, alſo having the Length of the Chord croſſing the Diameter in the given Part.

IN *Fig.* X. let FE be the Part of the Diameter given, alſo let AB be the given Chord which cuts off the given Part of the Diameter; it is required to find the whole Diameter Arithmetically.

FE the Part of the Diameter given, is 2 Feet and $\frac{7}{12}$ of an Inch, AB the Chord interſecting it is 9 Feet 10 Inches $\frac{10}{12}$ of an Inch.

The Rule is thus: Square one half of the Chord, and divide the Product by the Part of the Diameter given, which Quotient being added to the ſaid Part, is the Length of the Diameter required.

Example.

The whole Chord AB is 9 Feet 10 Inches 10 Parts, the Half whereof is 4 Feet 11 Inches 5 Parts, the Square of which is 24 Feet 6 Inches 2 Parts and 4 3ds, which being divided by 2 Feet 7 Parts (the Part of the Diameter given) the Quotient is 11 Feet 11 Inches 8 Parts, to which 2 Feet 7 Parts being added, make 14 Feet for the Length of the Diameter required.

PROP. XXIV. *Fig.* X.

Any Segment of a Circle being given, whoſe Chord-Line doth not exceed the Chord of the Quadrant of the ſame Circle; to find the Content without finding the Diameter, and without deſcribing any more of the Circumference.

ALthough, by the precedent Prop. XXII. any Segment of a Circle may be meaſured; yet becauſe the finding of the Center, and meaſuring of the Sector, and then meaſuring the Triangle and ſubtracting it from the Sector, requires a great deal of Trouble and Time; I will here ſhew how to do it with leſs Trouble, and very near the Truth, in ſmall Segments.

Let the Segment given be the ſame (whoſe Content was found by 22 Prop.) in *Fig.* X. whoſe Chord-Line A B is 9 Feet 10 Inches and 9 Parts, and the Part of the Diameter F E (cut off by the Chord Line, is 2 Feet 7 Parts and 6 Seconds; the Content of which Segment A F B E is required.

Note, If the verſed Sine, *viz.* the Line F E be not given, you muſt divide the Chord A B in the Middle, and from that middle Point, as at E, raiſe a Perpendicular to the Arch-Line.

Then take the whole Length of the Chord A B, and two Thirds of the Length of the Line F E, to which two Thirds add 7 Parts, then multiply thoſe two Lengths, and the Product gives you the Content.

Example.

The verſed Sine F E is 2 Feet 7 Parts 6 Seconds, two third Parts whereof is 1 Foot 4 Inches 5 Parts, to which 7 Parts being added, makes it 1 Foot 5 Inches; which multiply'd by the Chord A B, (namely) 9 Feet 10 Inches 9 Parts, produces 14 Feet and 2 Parts for the Content, which is 1 Part leſs than the Content found by the 22 Prop.

Another Example.

In *Fig. XI.* the Semi-diameter is 14 Feet, and the Content of the Quadrant A B C D will be found by the former Rules, according to *Metius*'s Proportion, to he 153 Feet 11 Inches 1 Part 6 Seconds.

Now it is required to find the Content of the Segment A B C E.

By ſubtracting the Content of the Triangle A C D, which is 98 Feet 4 Parts and 5 Seconds, from the Content of the Quadrant which is as above, there remains 55 Feet 10 Inches 9 Parts for the Content of the Segment A B C E.

Let us ſee what the other Way will produce.

The Length or Part of the Diameter B E is 4 Feet 1 Inch 2 Parts, two third Parts whereof is 2 Feet 8 Inches 9 Parts 4 Seconds, to which add 7 Parts, and it makes 2 Feet 9 Inches 4 Parts 4 Seconds; which being multiplied by the Chord-Line A C, whoſe Length is 19 Feet 9 Inches 7 Parts, the Product is 54 Feet 11 Inches 6 Parts for the Content of the Segment.

A Third Example.

Let the Content of the two little Segments, A B, B C, in *Fig. XI.* be required.

The Line FG is 1 Foot, two third Parts whereof is 8 Inches, to which 7 Parts being added, makes it 8 Inches 7 Parts, which being multiplied by 10 Feet 10 Inches, the Length of the Chord-Line A B produces 7 Feet 8 Inches 11 Parts, for the Content of the Segment AFBG, the other Segment BC being equal to this, add 7 Feet 8 Inches 11 Parts to the Content of the firſt, and the Product is 15 Feet, 5 Inches 10 Parts for the Content of both the little Segments.

The Triangle ABCE, whoſe Content is 40 Feet, 6 Inches 5 Parts, being deducted from the Content of the Segment ABCE, which is 55 Feet 10 Inches 9 Parts, there remains 15 Feet 4 Inches 4 Parts, for the Content of the two little Segments; which is half an Inch leſs than was found before.

PROP. XXV. *Fig. XII.*

An irregular Plat or Figure being given; to find the Area or Content thereof.

LET a b c d e f g h i in *Fig. XII.* be an irregular Plat, whoſe ſuperficial Content is required.

Reduce the ſame into as many Trapezia's as it will contain, at firſt the Trapezium a b c d; Secondly, a d e f; Thirdly, a f g, i, and there remains the Triangle, i g h. In which three Trapezia's, draw the Diagonals b d, d f, and f i, which ſhall be as common Baſes to each Triangle on either Side; on which Baſes let fall the Perpendiculars from the ſeveral Angles at a, c, e, and g; then in every Trapezium take the Length of the Baſe by itſelf, and the Length of the two Perpendiculars thereon falling, joined together, in one Number by itſelf; then multiply one half of the one in the whole of the other, and the Product is the Area of that Trapezium, which reſerve by itſelf, and working in like ſort with the reſt; and laſtly, the Triangle i, g, h: Collect all their Products together, which ſhall ſhew the Content required.

PROP. XXVI.

To find the Superficial Content of any Oval.

TAke both the Diameters, and multiply them one into another; then extract the Square Root of that Product, and that Root will be the Diameter of a Circle, whoſe Content will be equal to the Content of the Oval. For the Area of a Circle is to the Area of an Ellipſis, as the Square of the Diameter of that Circle is to the rectangled Figure of the tranſverſe and conjugate Diameters of the Ellipſis by the 6th of *Archimedes*'s Conoides and Spheroides.

Example.

Let *Fig. XIII.* be an Oval, whoſe longeſt Diameter is 10 Feet, and the ſhorteſt Diameter is 6 Feet, and the Content required.

Multiply the Length 10 by the Breadth 6, the Product is 60, the ſquare Root whereof is 7 Feet 8 Inches 11 Parts 5 Seconds, which is the Length of the Diameter, the Content of whoſe Circle is equal to the Content of the Oval; then multiply the Diameter by 10 Feet 7 Inches 7 Parts 5 Seconds (as you were ſhewn at the 5th Example of Prop. XVII. of this) and the Product is 82 Feet 4 Inches 6 Parts 3 Seconds, which divide by 12, it produces 6 Feet 10 Inches 4 Parts 6 Seconds 3 Thirds, which being multiplied by it ſelf (*viz.* ſquared) the Product is 47 Feet 1 Inch 5 Parts 11 Seconds, the Content of the Oval required.

See the whole Work.	F.	I.	P.	S.	T.
The Length of the Oval	10	0	0	0	0
The Breadth of the Oval	6	0	0	0	0
Being multiplied, the Product is	60	0	0	0	0
The Square Root whereof is	7	8	11	5	
Which being the M. Diam. mult. by	10	7	7	5	
	70	4			
	6	8	8	5	5
	4	1	6	6	
		9	2	3	4
		4	1	2	11
			4	8	2
			4	2	4
			2	11	
The Product is	82	4	6	3	2

Which Product divide by 12.

		F.	I.	P.	S.	T.
(1 Inch.	And it is	6	10	4	6	3
2 (0	Which multiply by	6	10	4	6	3
~~8~~2 (6 Feet						
~~1~~2		36	8	4	1	4
		5	2	3	4	2
		5	2	3	4	2
				3	5	3
				3	5	2
					1	6
					1	6
						2
The Content of the Oval is		47	1	5	11	3

In Decimals thus: Multiply the Length of the Oval by the Breadth, and divide the Product by 1.27324 Foot, it gives the Content.

PROP. XXVII. *Fig. XIV.*

To find the Superficial Content of a Cylinder, the Diameter being given.

NOte, That a Cylinder is a ſolid Body, which may well be repreſented by a Roll either of Timber or Stone, ſuch as are uſed in Gardens for the rolling of Walks.

The Proportion is, *As* 7 *to* 22, or *As* 113 to 355; *So* is the Diameter and Length of the Side multiplied one by the other, *To* the ſuperficial Content of the Outſide of the Cylinder, beſides the two Baſes or Ends.

The Rule thus: Multiply the Diameter by the Length or Side; then multiply that Product by 355, the Product whereof divide by 113, and it gives you the Content of the round Superficies.

Let *Fig. XIV.* be a Cylinder, whoſe Diameter is 4 Feet, and the Side or Length 10 Feet, and the Content required.

Firſt, Multiply the Diameter 4 by the Length 10, the Product is 40; which multiply by 355, and the Product is 14200; which divide by 113, and the Quotient 125 Feet, and 75 remaining, which is $\frac{75}{113}$ of a Foot; and being reduced, is 125 F. 7 I. 11 P. 6 S. for the Content required.

And if you had wrought after the Proportion of 7 to 22, the Content would have been 125 F. 8 I. 7 P. *fere*, which is a ſmall matter more than the former.

If you have the Circumference and Length of a Cylinder given, and the Content required;

Multiply the Circumference by the Length, and the Product is the ſuperficial Content.

Example.

Let the Circumference and Length of the Cylinder, *Fig. XIV.* be given, and the Content required.

Multiply the Circumference 12 F. 6 I. 9 P. 7. S. by the Length 10 F. and the Product is 125 F. 7 I. 11 P. for the Content required.

If you are minded to add the ſuperficial Content of the two Baſes or Ends of the Cylinder, you will find the Content of them as you are ſhewn before in meaſuring of Circles.

PROP.

PROP. XXVIII. *Fig. XV.*

To find the Superficial Content of a Cone.

A Cone is a Body which hath a Circle for its Baſe, from whence it diminiſheth equally (like a round Spire of a Steeple) till it end in a Point.

Let *Fig. XV.* be a Cone, the Circumference of whoſe Baſe is 24 F. and the Length of the Side 10 F. and the Content required.

If you require the Content of the whole Cone, that is to ſay, the Content of the Baſe, as well as the Content of the Outſide; then multiply the whole Side by half the Compaſs or Periphery of the Baſe; and to the Product add the Content of the Plain of the Baſe.

Example.

The whole Circumference of the Baſe being 24 F. the half thereof is 12 F. which being multiplied by the Length 10 F. produces 120 F. for the Content of the Outſide, beſide the Baſe: Then by the 13th Prop. of this, find the Content of the Circle whoſe Circumference is 24, and add it to the former Content, and the Product is the ſuperficial Content of the Cone, together with its Baſe.

Example.

	F.	I.	P.	S.
The Cont. of the Cone without the Baſe is	120	0	0	0
The Content of the Baſe is	45	9	9	9
The Content of the Cone, together with its Baſe, is	165	9	9	9

PROP. XXIX. *Fig. XVI.*

To find the Superficial Content of a Pyramid.

AS a Cone hath a round Baſe, and diminiſheth equally till it end in a Point; ſo a Pyramid hath an angular Baſe of 3, 4, 5, 6, or any Number of Sides, and diminiſheth to a Point at the Top.

Let *Fig. XVI.* be a Pyramid, whoſe Baſe is a Square, whoſe Side is 5 Feet, and the Length from the Baſe to the Top 10 Feet, and the ſuperficial Content required.

Add the four Sides of the Baſe together, of which Product take half; then multiply that half by the Side or Length.

Example.

One Side being 5 Feet, the four Sides added make 20 Feet, the Half whereof is 10 Feet; which multiplied by 10 Feet the Length, produces 100 Feet for the ſuperficial Content, beſides the Baſe: If you are minded to add the Content of the Baſe to it, which is 5 times 5 Feet, or 25 Feet, then the ſuperficial Content of the Pyramid, together with the Baſe, will be 125 Feet.

Likewiſe, if the Baſe be Triangular, you muſt add the three Sides of the Baſe together, and take half of the Product, which multiplied by the Length, gives you the Content: Then if you would add the Content of the Baſe to the Content of the Outſide, meaſure it as you are ſhewn before in meaſuring of Triangles.

Alſo if the Baſe be a Pentagon, take half the Product of the five Sides of the Baſe (being added together) and multiply it by the Length, and the Product gives you the Content: If you are minded to add the Baſe, find the Content, as you are ſhewn at Prop. IX. of this.

PROP. XXX.

To find the Superficial Content of a Globe or Sphere.

MUltiply the Diameter or Axis by the Circumference, and the Product will be the ſuperficial Content of the round Body or Globe.

Example.

Let the Diameter be 14 Feet, the Circumference will be 44 *fere*, which being multiplied one by the other, the Product is 616 Feet for the Content of the Superficies of the Globe.

Or find the Content of the Circle that hath the ſame Diameter as the Globe, and multiply that Content by 4, and the Product is the ſuperficial Content required.

Example.

A Circle whoſe Diameter is 14, the Content is 154, which multiplied by 4, the Product is 616, as before.

PROP.

PROP. XXXI. *Fig. XVII.*

To find the Superficial Content of a Fragment, or Part of a Globe.

THE Superficie of any Fragment, or Portion of a Globe, is to the Superficie of the remaining Portion, as the Portion of the Axis or Diameter cut off, is to the remaining Portion of the ſame.

Example.

Let there be in *Fig. XVII.* a Portion of a Globe given ABC, whoſe Part (or Segment) of the Axis (or Diameter) is 5 Feet, and the whole Diameter 14 Feet, and the ſuperficial Content of the Circular Part is required.

Say, by the Rule of Three, If 14 give 616, what will 5 give? Multiply 616 by 5, the Product is 3080; which divide by 14, and you have 220 Feet for the ſuperficial Content of the Segment, or Part of the Globe.

I ſhall now conclude the Meaſuring of Plains, *with ſome Directions whereby to meaſure* Land.

Wherein Note, That 5½ Yards, or 16½ Feet in Length, is called (according to Statute) a *Pole* or *Perch*; and one Perch in Breadth, and 40 in Length, is called a *Rood*; and 4 Perches in Breadth, and 40 in Length, is one *Acre*: So that an Acre contains 160 ſquare Perches, half an Acre 80 Perches, and a quarter of an Acre, commonly called a Rood, 40 Perches.

In meaſuring of Land, it is uſual to take the Lengths and Breadths with a Chain, the Length of which Chain ſome make to contain 4 Poles, (or Perches) as Mr. *Gunter* and others; and ſome make it to contain 2 Poles, as Mr. *Rathborn*, and others.

The Chain which I would adviſe is thus:

To contain 2 Perches (or 33 Feet) in Length, each of which Perches, divide into 12 equal Parts, or (as I may call them) Links, and each Link into 12 equal Parts, with little Wyers or Notches croſs-wiſe.

Thus the Length of a Perch will be divided into 144 Parts, and the whole Chain, conſiſting of 2 Perches, will contain 288 Parts; in the Middle of which Chain it will be convenient to have a pretty large Ring, that ſo you may diſtinguiſh where one Perch ends, and the other begins: Alſo midway between that large Ring and one end of the Chain hang a Curtain-Ring; and likewiſe hang another Curtain-Ring midway between the other end and the large Ring: So the whole Chain will be divided into four equal Parts or half Perches by the three Rings.

The Chain being thus divided, you have every whole Pole equal to 12 Links, or 144 Parts; every $\frac{3}{4}$ of a Pole equal to 9 Links, or 108 Parts; every half Pole equal to 6 Links, or 72 Parts: And laſtly, every quarter of a Pole equal to 3 Links, or 36 Parts.

The Chain being divided after this manner, you have only three Denominations to ſet down, namely, *Chains*, *Links*, *Parts*, if you will meaſure ſo near the Truth; but in moſt Menſurations you need proceed no farther than to Chains and Links, and then you have but two Denominations to ſet down.

Alſo the Manner of caſting up the Dimenſions, is the ſame as is ſhewn all along before, only altering the two firſt Denominations, to wit, whereas you did ſet over the Dimenſions F. I. P. *&c.* ſignifying Feet, Inches, Parts; ſo in meaſuring of Land, you muſt ſet over the Dimenſions C. L. P. ſignifying Chains, Links, Parts: For as a Foot contains 12 Inches, and an Inch 12 Parts, Line-Meaſure (or in Length) ſo likewiſe a Pole or Chain contains 12 Links, and one Link 12 Parts.

You muſt remember to double the Number of your Chains when you ſet them down, becauſe a Chain contains 2 Poles or Perches.

Suppoſe in *Fig.* V. CFEB to be a Piece of Land lying in form of a long Square, and the Length FE being meaſured with the Chain, to contain 8 Chains, 6 Links, and 6 Parts; becauſe the double of 8 is 16, I ſet it down thus . . . . . Alſo the Breadth EB ſet thus, containing

	C.	L.	P.	
	16	6	6	
	14	9	3	
	64	4	6	
Which I multiply as if they were Feet, Inches, and Parts	167	7	1	6
	12	4	4	6
				1
And the Product in ſquare Perches, is	244	4	0	1

Which 244 Chains or Perches you muſt divide by 160 to bring them into Acres, as you read above, that an Acre contains 160 ſquare Poles or Perches.

$$\begin{array}{l} (84 \\ \cancel{2}\cancel{4}\cancel{4}(1 \\ \cancel{1}\cancel{6}\cancel{0} \end{array}$$

So you have 1 the Quotient, which is 1 Acre, and the 84 that remains being Perches, divide by 40, the Number of Perches in a Rood, and you have the Quotient 2 Roods and 4 Perches remaining.

$$\begin{array}{l} (4 \\ \cancel{8}\cancel{4}(2 \\ \cancel{4}\cancel{0} \end{array}$$

Thus the Content of that Piece of Land

is found to be 1 Acre, 2 Roods, 4 Perches, and the 4 Links which remain in the Multiplication is one third Part of a Perch.

Again,

Suppoſe *Fig.* VII. to be a Field incloſed with three Sides, like a Triangle EDF, and you require to know the Content thereof;

Let the Side ED be 120 Chains or Perches, the Side DF 160 Chains or Perches, and the Side FE 200 Perches, and the Perpendicular DG 95 Perches.

As you were ſhewn in meaſuring of Triangles, take half the Baſe or Side EF, which is 100 Perches, and multiply it by the Perpendicular DG 95 Perches.

Example.

	C.	L.	P.
The Length of half the Baſe EF, is	100	0	0
Which multiplied by the Perpendicular	95	0	0
	500		
	900		
The Product is, in ſquare Perches	9500	0	0

Which being divided by 160, produces 59 Acres, 1 Rood and an half; (for if you divide 60 by 40, it is one and a half) which is the Content of that Triangular Piece of Land.

(6
~~150~~(0
~~9500~~(59
~~1600~~
~~16~~

Example.

Example.

A ſquare Piece of Land being in Breadth 120 Chains, 9 Links, 6 Parts; and in Length 125 Chains, 6 Links, and 2 Parts, and the Content is required.

	C.	L.	P.
The double of 120 is 240, ſet thus	240	9	6
The double of 125 is 250, ſet it thus	250	6	2
	12000	4	6
	480	125	1
	120		3
	187	6	
		40	
The Product in Perches is	60321	7	10

Which divide by 160, and the Quotient is 377 Acres, and 1 remaining, which is 1 Perch; to which add the 7 Links that are in the Product of the Multiplication, and the whole Content will be 377 Acres, 1 Perch, and 7 Links, which is half a Perch, and 1 Link.

The 10 Parts being but of a ſmall Value, are ſeldom ſet down.

$$\begin{array}{cccccl} & \cancel{1} & \cancel{1} & & & \\ \cancel{1} & \cancel{2} & \cancel{3} & \cancel{2} & (1 & \\ \cancel{6} & \cancel{0} & \cancel{3} & \cancel{2} & \cancel{1} & (\ 377 \\ \cancel{1} & \cancel{6} & \cancel{0} & \cancel{0} & \cancel{0} & \\ & \cancel{1} & \cancel{6} & \cancel{6} & & \\ & & \cancel{1} & & & \end{array}$$

Foraſmuch as in ſome Countries they allow 18 *Feet to the Pole or Perch, and in other Countries more, as* 20 *or* 24; *and all allow* 160 *of their Perches to an Acre: It will be convenient here to ſhew how to reduce one kind of Meaſure into another.*

Example.

Suppoſe it were required to reduce 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure (that is, 16 Feet and an half to the Pole) into cuſtomary Meaſure of 18 Feet to the Pole.

Firſt, reduce your given Quantity, 4 Acres, 3 Roods, and 4 Perches, into the leaſt Denomination, to wit, Perches; by multiplying the 4 Acres, by 160 (the Number of Perches in one Acre) and the Product is 640 Perches in the 4 Acres: Then multiply the 3 Roods by 40 (the Number of Perches in one Rood) and the Product is 120 Perches in the 3 Roods. Laſtly, add the 640 Perches, and the 120 Perches, alſo the 4 Perches together, and the total Product is 764 Perches, being contained in the given Number, *viz.* 4 Acres, 3 Roods, and 4 Perches.

Secondly, ſquare the two Poles or Perches, that is to ſay, multiply each in itſelf: As firſt, multiply the Pole of 18 Feet by 18 Feet, and the Product is 324 Feet: Then multiply the Pole of 16 Feet and an half, by 16 Feet and an half, and the Product is 272 Feet and 3 Inches.

Thirdly, ſay by the Rule of Three: *As* 324 (which is the Square of the 18 Feet Pole) *is* to 272 Feet and 3 Inches (which is the Square of the 16 Feet and an half Pole) *ſo* is 764 Perches (the Number given) *to* a fourth Number required.

Firſt, multiply 764 by 272 Feet and 3 Inches, and the Product is 207999, which divide by the firſt Number in the Queſtion, namely, 324, and the Quotient is 641 Perches, $\frac{315}{324}$ of a Perch, which being abbreviated, is $\frac{35}{36}$ of a Perch.

Fourthly, to reduce the Perches into Acres, divide them by 160; ſo 641 Perches being divided by 160, you find 4 Acres and 1 Perch remaining.

Thus you ſee, that 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure, of 16 Feet and a half to the Pole, being reduc'd, is 4 Acres, 1 Perch, and $\frac{35}{16}$ of a Perch cuſtomary Meaſure of 18 Feet to the Pole.

An Example of the whole Work.

The Quantity given is 4 Acres, 3 Roods, 4 Perches of Statute Meaſure, to be reduced to Cuſtomary Meaſure of 18 Feet to the Pole or Perch.

Firſt reduce the Quantity given into the leaſt Denomination, namely, Perches.

First reduce the 4 Acres.
The Perches in 1 Acre is 160; being multiplied,

The Perches in 4 Acres is 640

Secondly, Reduce the 3 Roods.
The Perches in 1 Rood is 40; being multiplied,

The Perches in 3 Roods is 120

The third Number 4 being the leaſt Denomination, namely, Perches, needs no reducing.

Now add the Perches together, 640
120
4

And you find the Quan. given contains 764 Perches.

The Quantity given being brought into Perches, the enſuing Work will be to find the Number of Superficial Feet that each Pole contains.

And firſt the Pole of	18 Feet.
Being ſquared, or multiplied by	18
	144
	18
Contains, or the Product is	324 Feet.

	F.	I.
Secondly, The Pole of	16	6
Being ſquared, or multiplied by	16	6
	96	
	168	
	8	3
Contains, or the Product is	272	3

Thirdly, by the Rule of Three ſay, *As* 324 Feet *is to* 272 Feet 3 Inches, *ſo* is 764 Perches *to* a Fourth Number required.

	F.	I.
Multiply the Second Number	272	3
By the Third	764	
	1088	
	1632	
	1904	
	191	
The Product is	207999	

Which divide by the firſt Number 324, and the Quotient is 641 Perches and $\frac{311}{324}$ Parts of a Perch, which abbreviated is $\frac{35}{36}$ Parts of a Perch, which is almoſt a Perch.

```
      (3
     6(1
   1353(5
  207999(641
   32444
    322
     3
```

Fourthly, Having now reduced the Quantity of given Perches (namely, 764 of Statute Meaſure) to 641 Perches, and $\frac{35}{36}$ of a Perch of Cuſtomary Meaſure of 18 Feet to the Pole or Perch; the next Work is to bring theſe Perches into Acres: Which to do, divide them by 160, and the Quotient (as you ſee in the Margin) is 4 Acres and 1 Perch; to which add the $\frac{35}{36}$ of a Perch, and then you have 4 Acres 1 Perch and $\frac{35}{36}$ of a Perch of Cuſtomary Meaſure (of 18 Feet to the Pole) contained in 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure, of 16 Feet and an half to the Pole.

```
     (1 Perch
  641(4 Acres.
  160
```

A Second Example.

Suppoſe, on the contrary, that 4 Acres, 1 Perch, and $\frac{35}{36}$ of a Perch of cuſtomary Meaſure of 18 Feet to the Pole, were required to be reduced into Statute Meaſure of 16 Feet and an half to the Pole.

As in the laſt Example, Firſt, reduce the given Quantity into the leaſt Denomination, to wit, Feet; for in this Example, the leaſt Denomination is Parts of a Perch, which are Feet.

Therefore firſt multiply the 4 Acres by 160, and the Product is 640, to which add the 1 Perch given, and it is 641 Perches.

Secondly, to bring theſe Perches into Feet, you muſt multiply them by the Number of Feet contained in a Perch; which as you found before in the laſt Example, by multiplying 18 by 18, the Product was 324: So then you muſt multiply 641 Perches by 324 Feet, and the Product will be 207684 Feet: To which you muſt add the Fraction $\frac{35}{36}$ whoſe Value you muſt find thus:

Firſt multiply the Numerator 35 by the Number of Feet in a Perch, to wit, 324, and the Product is 11340, which divided by the Denominator 36, and the Quotient is 315 Feet, which is the Value of the Fraction $\frac{35}{36}$, and muſt be added to the 207684 Feet and then the whole Number of Feet will be 207999. Thus you have the Quantity given reduced into the leaſt Denomination, namely, Feet.

And becauſe the Statute Pole 16 Feet and an half is a Fraction, and being ſquared, it produces a Fraction, and muſt be the firſt Number in the Queſtion, according to the Rule of Three, and by conſequence, the Diviſor of the Diviſion, which cannot be in Vulgar Arithmetick, to divide by an Integer, mix'd with a Fraction, altho' it may be done Decimally: We muſt therefore reduce the Feet into Inches, by multiplying 207999 by 12, the Product is 2495988 Inches; likewiſe multiply 272 Feet by 12, and the Product is 3264, to which add the 3 Inches that belong to the 272 Feet, and it is 3267 Inches.

Then divide 2495988 by 3267, the Number of Inches in a Statute Perch, and the Quotient is 764 Perches of Statute Meaſure, which divide by 160, and you have 4 Acres in the Quotient, and 124 Perches remaining; which 124 divide by 40, the Perches in a Rood, and you have 3 Roods and 4 Perches.

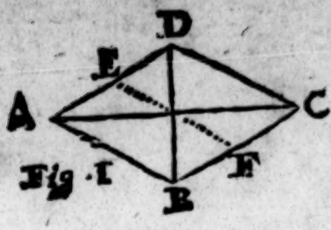


Fig ·I

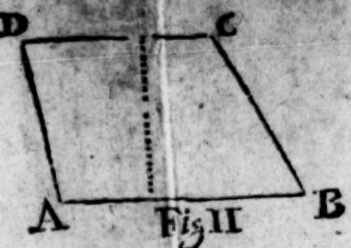


Fig II

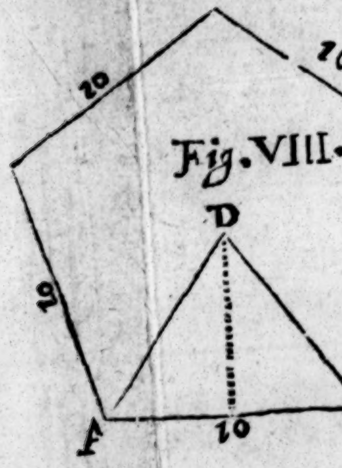


Fig.VIII

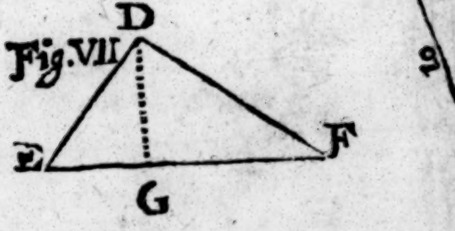


Fig.VII

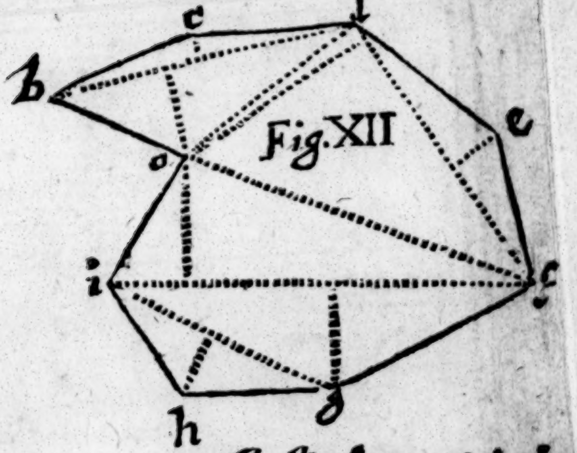
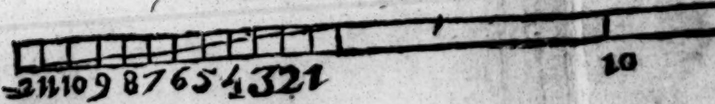


Fig.XII

F

The Scale appertaineing to theese



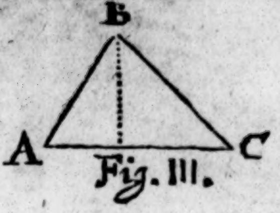


Fig. III.

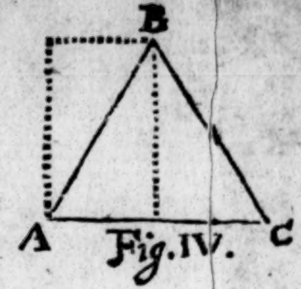


Fig. IV.

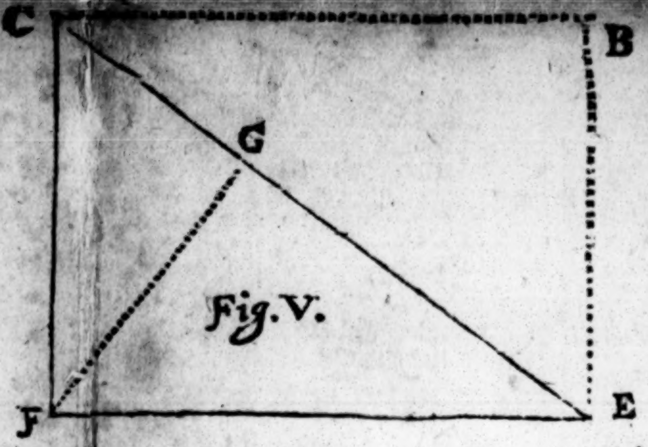


Fig. V.

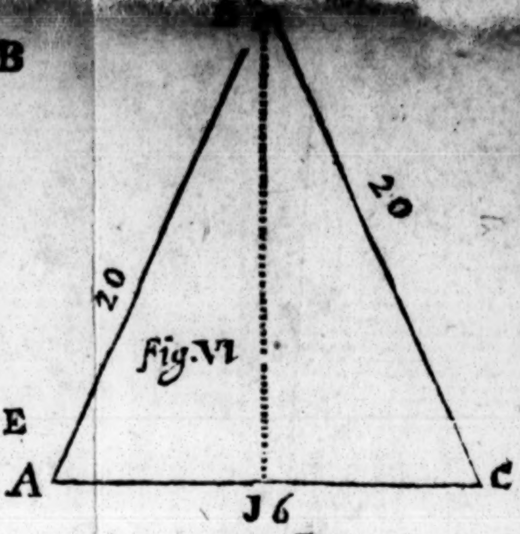


Fig. VI

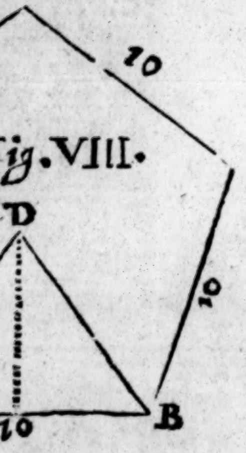


Fig. VIII.

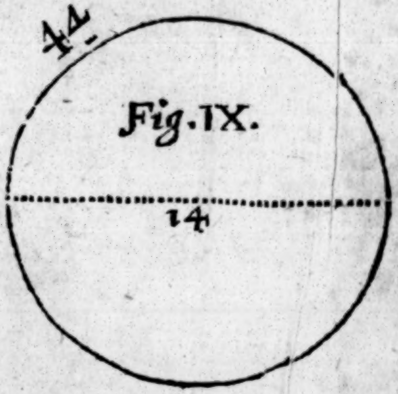


Fig. IX.

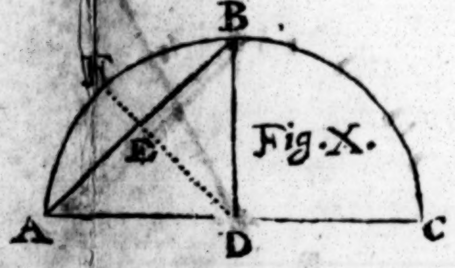


Fig. X.

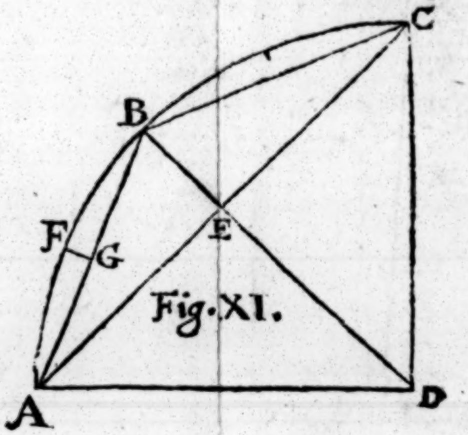


Fig. XI.

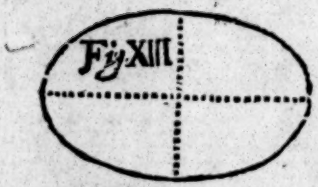
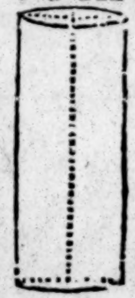


Fig. XIII

Fig. XIV


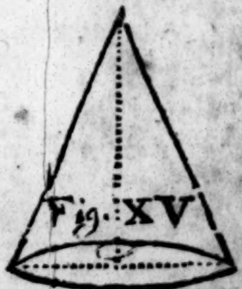


Fig. XV

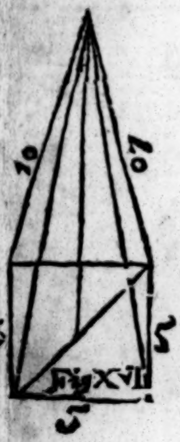


Fig. XVI

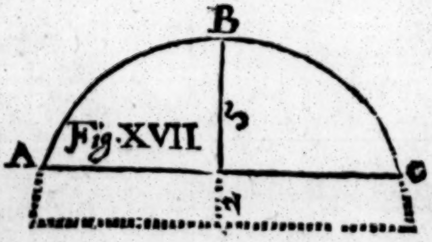


Fig. XVII

theese Figures

Place this at the End of the Fifth Book.

So that 4 Acres, 1 Perch, and $\frac{35}{36}$ of a Perch of Cuſtomary Meaſure, being reduced to Statute Meaſure, contains 4 Acres, 3 Roods, and 4 Perches, which was required to be done.

After the ſame Method may any Cuſtomary Quantity whatſoever be reduced.

I ſhall not need to mention any more Examples of Land that is circular, or irregular Plats, having already ſhewn how to find their Contents.

Note, When you meaſure the Length or Breadth of a Field with your Chain, you muſt meaſure in a ſtrait Line as near as you can, otherwiſe you will make the Length longer than it is.

I ſhall conclude the meaſuring of ſuperficial Plains with this Advice:

If you intend to learn any thing that is herein contained, you muſt not only read, but likewiſe put Pen to Paper, and caſt up the Examples, and alſo other Examples of your own propoſing, and with a little Uſe, mix'd with Diligence, you will ſoon attain to your Deſire; and you will find this Way of multiplying Fractions into Integers very pleaſant and brief, in reſpect of the other Way in Vulgar Arithmetick, whereby the Integers muſt be reduced into the leaſt Denomination, which is 4 times more Labour than this Way.

The End of the FIFTH BOOK.

MEASURING

OF

SOLIDS.

The SIXTH BOOK.

By *VEN. MANDEY.*

LONDON:
Printed in the YEAR M.DCC.XXVII.

MEASURING OF SOLIDS.

BY Meaſuring of Solids, is meant the Meaſuring of Stone, Timber, digging of Earth, and all ſolid Magnitude whatſoever.

And as from the Motion of a Line is made a Superficie; ſo likewiſe from the Motion of a Superficie, is made a Solid or Mathematical Body.

And as a Superficie conſiſts of Length and Breadth, ſo a Solid or Mathematical Body conſiſts of Length, Breadth, and Depth (or Thickneſs.)

There is no farther Progreſs; for which Way ſoever a ſolid Body is moved, a ſolid Magnitude will be thereby deſcribed.

In meaſuring of Solids, I ſhall begin with the Cube, it being moſt eaſy to meaſure.

A Cube is a Rectangular or Square Solid, that hath an equal Length, Breadth, and Depth, and is comprehended under ſix equal Squares, and may well be repreſented by a Dye, which is itſelf a little Cube.

And as in meaſuring of Plains, you only multiply the Breadth by the Length, for the gaining of the Content;

So in meaſuring of Solids, you muſt firſt multiply the Breadth by the Length, for the gaining the Superficial Content; and then multiply that ſuperficial Content by the Depth or Thickneſs of the ſolid Body, and the Product is the ſolid Content.

Of Timber or Stone being ſquare and equal-ſided.

PROP. I. *Fig.* I.

Fig. I. *being a Cube* 12 *Inches in Length, and* 12 *Inches in Breadth, and* 12 *Inches in Depth; what is the ſolid Content thereof?*

FIRST multiply 12 Inches the Length, by 12 Inches the Breadth, and the Product is 144 Inches for the ſuperficial Content; which multiplied by 12 Inches the Depth, produces 1728 Inches for the ſolid Content of that Cube.

Example.

	Inches.
The Length	12
The Breadth	12
	24
	12
The ſuperficial Content is	144
Which multiply by the Depth	12
	288
	144
And the ſolid Content required, is	1728

Where by the Way, you may take Notice, That as a ſuperficial Foot contains 144 ſuperficial Inches, ſo likewiſe a cubical or ſolid Foot contains 1728 cubical Inches, being produced as above.

So that by a Foot of Timber or Stone, or any other ſolid Material, in whatſoever kind of Solid it be found, is underſtood a Cube containing 1728 cubical Inches, and conſequently half a Foot Solid contains 864 cubick Inches, and a quarter of a Foot contains 432 cubick Inches.

PROP. II.

If the Side of a Cube of Timber or Stone be 4 Feet, 5 Inches, what is the Content of that Cube?

FIRST find the Content of the Side, by multiplying 4 Feet, 5 Inches by itſelf, that is, by 4 Feet, 5 Inches,

†

ches, it produces 19 Feet, 6 Inches, 1 Part for the ſuperficial Content of the Side; which Product multiplied by the ſaid 4 Feet, 5 Inches, produces 86 Feet, 1 Inch, 10 Parts, and 5 Seconds, for the Content required.

Example.

	F.	I.	P.	S.	T.
The Side given is	4	5	0	0	0
Which multiply by itſelf	4	5	0	0	0
	16	2	1		
	1	8			
	1	8			
The Superficial Content of the Side is	19	6	1		
Which multiply by the Depth or Side	4	5			
	76	2	6	5	
	2				
	7	11	4		
The ſolid Content required, is	86	1	10	5	

95

PROP.

PROP. III. *Fig.* II.

Suppoſe in Fig. II. ABCD *to be a ſtrait Piece of Timber, terminated at both Ends by two equal long Squares (which may be called the Baſes) the Length of which Piece* BD *is* 3 *Feet,* 4 *Inches, and the broader Side of the Baſe* AB 1 *Foot,* 8 *Inches, and the narrower Side* BC 1 *Foot,* 1 *Inch, and the Content of this Piece be required ;*

FIRST find the ſuperficial Content of the Baſe or End, by multiplying the broader Side 1 Foot, 8 Inches, by the narrower Side 1 Foot, 1 Inch, the Product is 1 Foot, 9 Inches, 8 Parts for the ſuperficial Content of the End; which 1 Foot, 9 Inches, 8 Parts, being multiplied by the Length 3 Feet, 4 Inches, produces 6 Feet, 2 Parts, 8 Seconds, for the ſolid Content of that Piece of Timber.

Example.

	F.	I.	P.	S.	T.
The broader Side of the Baſe AB, is	1	8	0	0	0
The narrower Side BC, is	1	1	0	0	0
Being multiplied.	1	8			
		1	8		
The ſupeficial Content of the Baſe is	1	9	8		
Which multiply by the Length AE	3	4			
	34				
	23				
	3	2	8		
	2				
The ſolid Content required, is	6	0	2	8	

PROP. IV.

Suppoſe a Piece of Timber or Stone, whoſe Length is 3 Feet 4 Inches, be terminated at both Ends by two equal Quadrats (or Squares) the Side whereof is 1 Foot 7 Inches; what is the Content of ſuch Piece?

FIRST find the Content of one End or Baſe, by multiplying 1 Foot 7 Inches, by 1 Foot 7 Inches, it produces 2 Feet 6 Inches, and 1 Part for the Content; then multiply the Content of the End by the Length 3 Feet 4 Inches, and it produces 8 Feet, 4 Inches, 3 Parts, and 4 Seconds, for the ſolid Content.

An Advertiſement.

The uſual Way among many Men to meaſure Timber, whoſe Sides are unequal (although it be a falſe Way) is this: They uſually add the Breadth and Depth together, and take the half for the Side of a mean Square; and after they have found the Content of that mean Square, as they call it (falſe Square as I term it) they multiply it into (or by) the Length, and the Product they conclude is the Content of the Piece.

Let us compare this falſe Way with the true Way.

Suppoſe the Piece of Timber in the preceding *Prop.* III. whoſe Sides are unequal, to wit, one Side of the Baſe (or End) AB, is 1 Foot 8 Inches, and the other Side of the ſame Baſe (or End) BC, is 1 Foot 1 Inch.

Example of the falſe Way.

Firſt they add the two Sides together, namely, 1 Foot 8 Inches, and 1 Foot 1 Inch, which make 2 Feet 9 Inches, the half whereof is 1 Foot 4 Inches and 6 Parts, which they multiply by itſelf, namely, 1 Foot 4 Inches and 6 Parts, and the Product is 1 Foot 10 Inches 8 Parts and 3 Seconds, which they alſo multiply by the Length 3 Feet 4 Inches, and the Product is 6 Feet, 3 Inches, 7 Parts, and 6 Seconds, for the Content, which is more than the true Content by above a quarter of a Foot, and the more unequal the two Sides are, the more falſe this Way of Meaſuring gives the Content.

Of Timber or Stone, conſiſting of 3, 5, or more Sides.

PROP. V. *Fig.* III.

Admit Fig. **III.** *to be a ſtrait Piece of Timber or Stone, whoſe Length* **AB** *is* 30 *Feet, to be terminated at both Ends with two equilateral Triangles, the Side whereof* **GF** *is* 12 *Feet; the Content whereof is required.*

Take this as a general Rule for regular Polygons.

FIRST meaſure all the Sides round about, or girt them with a Line; then take half that Length for one Sum or Breadth; then find the Center H of the Baſe, which you may do by finding the Middle of the Baſe Line, as alſo of one Side; and from the Angles oppoſite to the Baſe Line and that Side, draw two Lines to the Middle of them, and where thoſe Lines interſect, there is the Center of the Triangle, as in the Figure at H; then take the neareſt Diſtance from the Center to one of the Sides for the other Sum or Depth; then multiply that Breadth and Depth one by the other, and the Product is the ſuperficial Content of the End or Baſe of the Piece, which Content multiplied by the Length of the Piece, produces the ſolid Content.

Example.

The Girt of the 3 Sides is 36 Feet, the half whereof is 18 Feet, which multiply by the neareſt Space from the Center to one Side, which is 3 Feet, 5 Inches, 6 Parts, 10 Seconds, and the Product is 62 Feet, 4 Inches, and 3 Parts, for the Content of the Baſe or End of the Piece, which multiplied by the Length of the Piece 30 Feet, produces 1870 Feet, 7 Inches, and 6 Parts for the Content of the Piece.

The Operation.

	F.	I.	P.	S.	T.
The 3 Sides being added, is 36 Feet, the half whereof is	18	0	0	0	0
Which multiply by the neareſt Space from the Center, to wit,	3	5	6	10	0
	54	9	15		
	7	6			
The ſuperficial Content of the Baſe is	62	4	3		
Which multiply by the Length	30				
	1860	7	6		
	10	7			
The ſolid Content of the whole Piece is	1870	7	6		

Otherwiſe

Otherwiſe thus:

Find the Perpendicular of the Triangle by the 4th Prop. of meaſuring of Plains; then multiply the Perpendicular by half the Baſe, the Product whereof being multiplied into the Length, gives the ſolid Content.

Example.

	F.	I	P.	S.
The Perpendicular is	10	4	8	6
Which multiply by half the Baſe	6			
	60	4	3	
	2			
The Content of the End is	62	4	3	
Which multiply by the Length	30			
	1860	7	6	
	10			
The Content (as before) is	1870	7	6	

PROP. VI. *Fig.* IV.

Admit Fig. IV. *were a ſtrait Piece of Timber, terminated at both Ends with two equal Pentagons, whoſe Side* AB *is* 12 *Inches, and the neareſt Diſtance from the Center* C, *to the Middle of the Side at* D *is* 8 *Inches, and the Length* EF, 15 *Feet, and the Content required.*

Example.

	F.	I.	P.
THE 5 Sides added together is 5 Feet, the half whereof is	2	6	0
Which multiply by CD	0	8	
	1	4	
		4	
The Content of the Baſe is	1	8	
Which multiply by the Length	15		
	15		
	10		
The ſolid Content is	25		

Of

Of ſquared Timber or Stone, being bigger at one End than at the other, in which Form ſome Timber Trees grow; and being fell'd, are ſo hewn, and brought to a Square.

PROP. VII. *Fig.* V.

Admit Fig. V. *to be a ſtrait Piece of Timber or Stone, terminated at both Ends with two unequal Squares; the Side of the greater Square (or End)* AB, *is* 1 *Foot* 8 *Inches, and the Side of the leſſer Square (or Upper-end)* EF, *is* 1 *Foot* 3 *Inches, and the Length* AE 20 *Feet; what is the Content?*

The Rule is this:

1. FIND the Content of the lower End (or greater Square) ABC. 2. Find the Content of the upper End (or leſſer Square) EGDF. 3. Multiply one Content by the other. 4. From that Product extract the Square Root. 5. Add this Square Root, and the whole Content of both the Baſes into one Sum. 6. Multiply this Sum by one Third Part of the Altitude (or Length) of the Piece, and the Product will be the ſolid Content required.

Which Rule is demonſtrated by Mr. *Oughtred*, for meaſuring a Segment of a Pyramid, in *Ch.* 19. *Prob.* 21. of his *Clavis Mathematicæ.*

Example.

Firſt, Find the ſuperficial Content of the lower End ABC, by multiplying 1 Foot 8 Inches, by 1 Foot 8 Inches, it produces 2 Feet, 9 Inches, 4 Parts, for the ſuperficial Content of the lower End.

Secondly, Find the Content of the upper End (or leſſer Square) EGDF, by multiplying 1 Foot 3 Inches, by 1 Foot 3 Inches, the Product whereof is 1 Foot, 6 Inches, and 9 Parts, for the ſuperficial Coutent of the leſſer Square (or upper End.)

Thirdly, Multiply one Content by the other, to wit, 1 Foot, 6 Inches, and 9 Parts, by 2 Feet, 9 Inches, and 4 Parts, and the Product is 4 Feet, 4 Inches, and 1 Part.

Fourthly, Extract the Square Root of 4 Feet, 4 Inches, and 1 Part, which Root is 2 Feet and 1 Inch.

Fifthly, Add this Square Root and the Content of both the Baſes together, all three in one Sum, and the Product is 6 Feet, 5 Inches, and 1 Part.

Sixthly, Multiply the laſt Sum by a Third Part of the Length, namely, 6 Feet 8 Inches, and the Product will be 42 Feet, 9 Inches, 10 Parts, and 8 Seconds, for the ſolid Content required.

The

The Operation.

	F.	I.	P.	S
The Side AB is	1	8	0	0
Which ſquare or multiply by itſelf	1	8	0	0
	1	8		
		8		
		5	4	
The ſuperficial Cont. of the lower End is	2	9	4	
The Side of the leſſer Square, or upper End, is	1	3		
Which ſquare or multiply by itſelf	1	3		
	1	3	9	
		3		
The ſuperficial Cont. of the upper End is	1	6	9	
Which multiply by the Cont. of the lower	2	9	4	
The Product of one End multiplied by the other is	4	4	1	
The Square Root whereof is	2	1		
To which add the Cont. of the lower End	2	9	4	
Alſo add the Content of the upper End	1	6	9	
And the Product is	6	5	1	
Which multiply by 1 Third of the Leng.	6	8		
	36	3	4	
	2	6	6	
	4			8
The ſolid Content required is	42	9	10	8

An

An Advertiſement.

The uſual Way which a great many Men uſe in meaſuring ſuch a Piece of Timber or Stone, is this, which is a falſe Way, and finds a Content leſs than the true Content.

They take the Sides of the Square about the Middle of the Piece, and this Square of the Middle of the Piece they ſuppoſe to be a mean Square between the greater End and the leſſer End; then they multiply the Content of this Square by the Length of the Piece.

Let us compare this falſe Way with the true Way, and ſee what Difference there will be between the Content of the Piece found by the foregoing 7th *Prop.* and the Content of the ſame Piece found according to the uſual (but falſe) Way.

Example.

The Side of the Square taken in the Middle of the Piece is 1 Foot, 5 Inches, and 6 Parts, which multiplied by itſelf produces 2 Feet, 1 Inch, 6 Parts, and 3 Seconds, for the Content of the mean Square, as they account it; which Content they multiply by the Length of the Piece 20 Feet, and it produces 42 Feet, 6 Inches, and 5 Parts, for the ſolid Content which is above a quarter of a ſolid Foot leſs than the true Content found before; and the more the Tree (or Solid whatſoever) diminiſheth, the more the Content, meaſured this falſe Way, will want of the true Content.

Example.

Suppoſe a Piece of Timber, being the ſame Length 20 Feet, were 1 Foot 8 Inches Square, at the lower (or greater) End, and 1 Foot Square at the upper (or leſſer) End, and the Content required.

Firſt, We will find it the true Way.

	F.	I.	P.	S.
The Content of the greater End (whoſe Side is 1 F. 8 I.) is	2	9	4	0
The Content of the leſſer End (whoſe Side is 1 F.) is	1	0	0	0
Which being multiplied produces	2	9	4	
The Square Root whereof is	1	8		
To which add the Content of the greater End	2	9	4	
Alſo add the Content of the leſſer End	1	0	0	
The Product is	5	5	4	
Which mult. by ⅓ Part of the Length	6	8		
	30	3	4	
	2	6	2	8
	3	4		
		2		
The true ſolid Content is	36	3	6	8

Now let us ſee what the uſual (but falſe) Way will produce.

	F.	I.	P.
The Side of the Square taken in the Middle is	1	4	0
Which multiply by itſelf, namely,	1	4	0
	1	4	
		4	
		1	4
The Content of the mean Square is	1	9	4
Which multiply by the Length	20		
	20		
	15	6	8
The Content (the falſe Way) is	35	6	8

Which Content is leſs than the true Content by almoſt ¼ of a ſolid Foot, which in meaſuring of a great Quantity of Taper-grown Timber would be conſiderable.

Of Pyramids.

A *Pyramid* is a Solid, comprehended under plain Surfaces, and from a Triangular, Quadrangular, or any Multangular Baſe, diminiſheth equally leſs and leſs, till it finiſh or end in a Point at Top; ſuch are the Spires of ſome Steeples.

Note, If a Pyramid be cut into two Segments or Parts by a Plain parallel to the Baſe, one of thoſe Segments or Parts will be a Pyramid (to wit, that which diminiſheth to a Point) and the other Segment or Part will have two unequal Baſes, ſuch as is *Fig.* V. The manner of finding its Content is ſhewn at *Prop.* VII.

PROP.

PROP. VIII. *Fig.* VI.

Admit Fig. VI. *were a Pyramid, whoſe Baſe is an Equilateral Triangle, and one Side of the Baſe* AB *is* 1 *Foot, and the Perpendicular of the Baſe* CD 10 *Inches,* 4 *Parts,* 6 *Seconds, and the Length of the Pyramid* RS 8 *Feet; what is the Content of this Pyramid?*

The Rule for finding the Content, is this:

MUltiply the Content of the Baſe by one Third Part of the Height of the Pyramid, and the Product is the ſolid Content. (By Height is to be underſtood a Perpendicular Line, that falleth from the Top of the Pyramid, to the Middle or Center of the Baſe: You are not to underſtand by Height, the Length of the Slope-line on the Outſide RS, for that is longer than the Altitude or Height.) And before we proceed any farther, it will be convenient to ſhew how to find the Height of any Pyramid.

How to find the Height of any Pyramid.

Firſt, Square the Semidiameter of the Baſe.

Secondly, Square the Length of the Pyramid, which is the Slope-line RS.

Thirdly, Subtract the leſſer Square out of the greater.

Fourthly, Find the Square Root of the Remainder, and that is the Height required.

Example.

Firſt, The Semidiameter of the Baſe EF is 3 Inches, 5 Parts, 7 Seconds, which being ſquared, is 12 Inches (or 1 one of thoſe Inches whereof 12 make a ſuperficial Foot.)

Secondly, The Length of the Pyramid R S is 8 Feet, which being ſquared is 64 Feet.

Thirdly, The leſſer Square, 1 Inch being taken out of the greater Square 64 Feet, there remains 63 Feet and 11 Inches.

Fourthly, The Root of 63 Feet and 11 Inches being extracted, is 7 Feet, 11 Inches, 11 Parts; which is the Height of the Pyramid required.

And this Rule is general for the finding the Height of any Pyramid, whether Triangular, Quadrangular, or Multangular, always remembring to find the Center of the Baſe, that ſo you may have the Semidiameter thereof; which Centers are found by drawing Lines from the Angles, to the Middle of the Sides oppoſite to them.

The Operation.

	I.	P.	S.	T.	
The Semidiameter of the Baſe being E F, is	3	5	7	0	
You muſt ſquare, that is, mult. by	3	5	7		
		9	2	1	
		1	3	2	11
		1	3	2	11
			1	9	
			1	9	
The Square of the Semidiameter is	1	0	0	0	10

The

	F.	I.	P.	S.
The Length of the Pyramid RS is	8	0	0	0
Which ſquare or multiply by itſelf	8			
And the Length being ſquared, is	64	0	0	0
Out of which ſubtract the Square of the Semidiameter		1		
And there remains	63	11	0	0
The Square Root whereof (being the Height) is	7	11	11	11 +

Thus having found the Height of the Pyramid, we will proceed in the next Place, to find the Content of the Baſe.

	F.	I.	P.	S.	
The Perpendicular C D is	0	10	4	6	
Which multiply by half the Baſe AC		6			
The Content of the Baſe is	0	5	2	3 +	
Which multiply by $\frac{1}{3}$ of the Height	2	7	11	11	
		10	4	1	10
		2	11	6	2
			1	2	2
			4	7	
				1	9
				4	7
The ſolid Content of the Pyramid, is	1	1	9	11	6

PROP.

PROP. IX. *Fig.* VII.

Admit Fig. VII. *were a Pyramid, whoſe Length* AE *is* 15 *Feet, and the Baſe of it* ABCD *a Square, whoſe Side* AB, *or* CD, *is* 2 *Feet* 6 *Inches; what is the Content?*

FIRST find the Altitude or Height, which is the prick Line EF, by the Rule in the foregoing *Prop.* VIII. then multiply one Third Part of the Heighth by the Content of the Baſe ABCD, and the Product is the Content required.

The Operation.

	F.	I.	P.	S.	T
The Side of the Square of the Baſe AB, or CD, is	2	6	0	0	0
Which multiply by itſelf	2	6			
	4	3			
	1				
	1				
The Content of the Baſe is	6	3	0	0	0
Which multiply by $\frac{1}{3}$ of the Height	4	11	9	6	4
	24	2	9	3	
	1	6	2	1	6
	5	4	6	2	1
			3		
The ſolid Content of the Pyram. is	31	1	8	6	7

How

How to find the Third Part of any Number.

Suppoſe you were to find the one Third Part of the Height of the Pyramid aforeſaid.

	F.	I.	P.	S.	T.
Set down your Number thus	14	11	4	7	0
Then ſay, The $\frac{1}{3}$ Part of 14 Feet is	4	8			
The $\frac{1}{3}$ Part of 11 Inches, is		3	8		
The $\frac{1}{3}$ Part of 4 Parts, is			1	4	
The $\frac{1}{3}$ Part of 7 Seconds, is				2	4
Being added, the Product is	4	11	9	6	4

Which is the Third Part required.

After the ſame manner you may find the Third Part of any other Number; you may likewiſe find the Fourth Part or Fifth Part of any Number after the ſame manner.

PROP. X. *Fig.* VIII.

Admit Fig. VIII. *were a Pyramid, whoſe Length* AH *is* 15 *Feet, and the Baſe thereof a Pentagon, whoſe Side* AB *is* 2 *Feet*, 6 *Inches, and the Content required.*

FIRST find the Content of the Baſe ABCDE (by the Rule delivered at the 9th Prop. of *Book* 5. *Of Meaſuring of Plains*) which I will here repeat.

The Rule is this:

As 182 *is to* 125: *So* is the Side of the Pentagon given, *to* the Semidiameter of a Circle inſcribed within that Pentagon.

Then ſay, If 182 give 125, what will (the Side of the Pentagon) 2 Feet 6 Inches give?

	F.	I.
Multiply the Second Number	125	0
By the Third Number	2	6
	250	
	62	6
And the Product is	312	6

Which divide by the firſt Number 182, and the Quotient is 1 Foot, and 130 remains, which multiply by 12, and the Product is 1560, which divide by 182, and the Quotient is 8 Inches, and 104 remains; which multiply by 12, and the Product is 1248, which divide by 182; and the Quotient is 6 Parts, and 156 remaining; which multiply by 12, and the Product is 1872, which divide by 182, and the Quotient is 10 Seconds, and 52 remains; which multiply by 12, and the Product is 624, which divide by 182, and the Quotient is 3 Thirds, as you may ſee in the Margin: There is yet a

(130
$\cancel{3}\cancel{1}\cancel{2}$ (1 Foot.
$\cancel{1}\cancel{8}\cancel{2}$

130
12

260
130

1560

(104
$\cancel{1}\cancel{5}\cancel{6}\cancel{0}$ (8 Inches.
$\cancel{1}\cancel{8}\cancel{2}$

Remainder, but the Value of it is not $\frac{1}{1000}$ Part of an Inch; and in many Caſes you will not need to proceed any farther than to Parts of Inches.

```
 104
  12
 ----
 208
104
----
1248
```

In the next Place, there is the 6 Inches that belong to the 312 Feet, to be divided by 182; but becauſe the Diviſor is greater than the Dividend, multiply the 6 Inches by 144, and the Product is 864 Seconds; then divide the 864 by 182, and the Quotient is 4 Seconds, which you muſt add to the 10 Seconds, and they make 1 Part and 2 Seconds.

```
(156
1248 (6 Parts.
 182

 156
  12
 ----
 312
156
----
1872
```

So is there found the Length of the Semidiameter of the Circle inſcribed within the Pentagon, (whoſe Side was given) namely, 1 Foot, 8 Inches, 7 Parts, 2 Seconds, and 3 Thirds, which is the neareſt Diſtance from the Center of the Pentagon, to either of the Sides.

```
 0(5
1872 (10 Sec.
1822
 18
```

The next Work will be to find the Content of the Baſe of the Pyramid.

	F.	I.	P.	S.	T.
Firſt, add the 5 Sides togther, and they make 12 Feet and 6 Inches the half whereof is	6	3	0	0	0
Which multiply by the Semidiameter ſound, which is	1	8	7	2	3
	6	3	1	9	6
	4	2	6	1	6
		3	1		
And the Content of the Baſe ABCDE, is	10	8	8	11	0
Which multiply by a Third Part of the Height of the Pyramid	5				
	50		4	7	
	3	4			
		3	4		
And the Solid Content of the Pyramid, is	53	7	8	7	

I ſhall not inſtance any more Examples of Pyramids; for if the Baſe conſiſt of 6 or more Sides, the Content of the Baſe may be ſound by the Rules delivered in *Book* 5. and then by multiplying the Content of the Baſe into one Third Part of the Height of the Pyramid, the Product is the ſolid Content.

Of round Solids, ſuch as are Cylinders, Cones, Spheres, Columns, &c.

PROP. XI. *Fig.* IX.

Admit Fig. IX. *were a Cylinder (by Cylinder is meant ſuch a like Solid as is uſed in Gardens for the Rolling of Walks) whoſe Length* AB *is* 6 *Feet, and the Diameter* BC 1 *Foot and* 4 *Inches, and the Content required.*

The Rule for Meaſuring ſuch a Solid, is this:

MUltiply the Content of the Baſe BC, or AD, by the Length AB, and the Product is the ſolid Content.

Firſt let us find the Content of the Baſe, the Rule is this: *As* 452 (that is four times 113) *is to* 355; *So* is the Square of the Diameter *to* the Content required.

	F.	I.	P.	S.
The Diameter is 1 F. 4 I. the Square of it is	1	9	4	0
Which multiply by	355			
	355	118	4	
	266	3		
And the Product is	631	1	4	

Which divide by the firſt Number 452, and the Quotient is 1 Foot, as you ſee in the Margin, and 179 remains; then continuing multiplying the Remainders by 12, and dividing the Products by 452, till you come to the Denomination of Thirds, as you ſee in the Margin, at laſt your Quotient is 9 Parts, and 12 remaining, which 12 is inconſiderate in Value; but for the 1 Inch and 4 Parts that belong to the 631 Feet; and for the 12 that remain above your laſt Diviſion, you may add 6 Thirds; for if you ſhould continue multiplying and dividing the Remains, you would produce 6 Thirds.

(179
~~631~~ (1 Foot.
~~452~~

179
12
358
179
2148

(340
~~2148~~ (4 Inches.
~~452~~

340
12
680
340
4080

(12
~~4080~~ (9 Parts.
~~452~~

	F.	I.	P.	S.	T.
Then the Content of the Baſe is found to be	1	4	9	0	6
Which muſt be multiplied by the Length AB	6				
	6 2	4	6	3	
And the Product is	8	4	6	3	

Which is the ſolid Content of the Cylinder.

Of a Cone.

A *Cone* is a Solid, which hath a Circle for its Baſe, from whence it grows equally leſs and leſs (like a Sugar-Loaf) till it finiſh or end in a Point at Top.

PROP. XII. *Fig.* X.

Admit Fig. X. *were a Cone, whoſe Length* AC *is* 15 *Feet, and the Diameter of the Baſe* AB 1 *Foot* 4 *Inches; what is the Content thereof?*

THE Rule for finding the ſolid Content of a Cone, is the ſame as for the finding the ſolid Content of a Pyramid, to wit, by multiplying the Content of the Baſe by one Third Part of the Height.

Firſt find the Height of the Cone (according to the Rule preſcribed in *Prop.* 8. of this Book) for the Height of a Cone is found by the ſame Rule that the Height of a Pyramid is found.

	F.	I.	P.
1. The Semidiam. of the Baſe A B, is	0	8	0
Which ſquare (or multiply by itſelf)	0	8	0
The Square of the Semidiameter is	0	5	4

	F.	I.	P.	S.	T.
2. The Length of the Cone AC is	15				
Which ſquare (or multiply by itſelf)	15				
	75				
	15				
The Product is	225				
3. Out of the Square of the Length AC, ſubtract the Square of half the Diam. AB, to wit,		5	4		
And the Remainder is	224	6	8		
The Square Root whereof is	14	11	9	11 *fere*	
Which is the Height of the Cone; of which Height we muſt take one Third Part, which is	4	11	11	3	8

Having found the Height, and taken a Third Part thereof, the next Work will be to find the Content of the Baſe.

Then ſay, by the preceding Rule in *Prop.* XI. *As* 452 *is* to 355: *So* is the Square of the Diameter *to* the Content of the Baſe.

	F.	I.	P
The Square of the Diameter is	1	9	4
Which multiply by	355		
	355	118	4
	266	3	
And the Product is	631	1	4

Which

	F.	I.	P.	S.	T.
Which divide by 452, and you will find the Cont. of the Baſe to be. (as in *Prop.* XI.)	1	4	9	0	6
Which multip. by one Third Part the Height of the Cone	4	11	11	3	8
	4	11	11	3	5
	1	4	8	3	
		3	8	8	3
		3	3	8	2
				1	8
				2	2
The ſolid Content of the Cone is	6	11	8	2	8

Note, If a Cone be cut into two Segments (or Parts) by a Plain parallel to the Baſe; one of thoſe Parts will be a Cone, and the other Part will have two unequal Circles for the Baſes: As in *Fig.* XI. the upper Part is a Cone, and the lower Part is a Segment of a Cone, which hath two unequal Circles for the Baſes.

PROP. XIII. *Fig.* IX.

A Segment of a Cone may well be repreſented by a Tree that grows taper or diminiſhing.

ADmit the Segment in *Fig.* XI. were a round Taper Piece of Timber, whoſe Length AB is 15 Feet, and the Diameter of the lower Baſe AC 1 Foot 4 Inches; and the Diameter of the upper End, or Baſe DB, is 1 Foot; what is the Content of this Piece?

The Length A B being 15 Feet, and the Diameter AC being 1 Foot 4 Inches; the Length, and likewiſe the Diameter is the ſame as was the Cone in the laſt Propoſition.

For finding the Content of this Piece of Timber (or Segment of a Cone) you muſt have reſpect to the Rule delivered in the preceding 7th Prop. of this Book, for the ſame Rule that ſerveth for ſquare Timber, whoſe Baſes or Ends are unequal, ſerveth likewiſe for round Timber, or Bodies whoſe Baſes or Circles at the Ends are unequal.

Example.

The Content of the lower Baſe AC, is, by the laſt foregoing Propoſition found to be 1 F. 4 I. 9 P. 0 S. 6 T.

Now we muſt find the Content of the upper Baſe, or leſſer End BD, whoſe Diameter is 1 Foot.

And becauſe the Square of the Diameter is but 1 Foot, the Content of the Baſe cannot be ſo much as 1 Foot; therefore we muſt ſtate the Queſtion in Inches, ſaying thus:

If 452 give 355; what will the Diameter, whoſe Square is 12 Inches, give?

For, if you remember, the Rule is, *As* 452 *is* to 355: *So* is the Square of the Diameter, *to* the Content of the Circle of the Baſe (or End of the Piece.)

Therefore

Inches.

Therefore multiply the ſecond Number 355
By the third Number in the Queſtion, to wit, 12

710
355

And the Product is 4260

Which divide by the firſt Number 452, and the Quotient is 9 Inches, and $\frac{192}{452}$ of an Inch; which, when reduced, as is ſhewn before, by multiplying the Remainders by 12, and dividing the Products by 452, you will find 9 Inches, 5 Parts 1 Second, and 2 Thirds, for the Content of the upper End or Baſe of the Piece.

(192
4260 (9 Inches.
452

Having now the Content of both the Baſes, the next Work will be to multiply them one by the other.

	F.	I.	P.	S.	T.	Fo.
The Content of the leſſer Baſe BD, is	0	9	5	1	2	0
Which multiply by the Content of the greater Baſe AC	1	4	9	0	6	0
		9	6	9	2	9
		3	1	8	4	
			5	3	9	8
				1	4	6
						1
						2
The Product of the two Baſes multiplied, is	1	1	1	10	9	2

The Square Root whereof is 3 I. 7 P. 6 S.

Which Square Root, and the Content of both the Baſes, muſt be added together.

	F.	I.	P.	S.	T.
The Square Root is	0	3	7	6	0
The Content of the greater Baſe is	1	4	9	0	6
The Content of the leſſer Baſe is	0	9	5	1	2
And the Product of the Addition is	2	5	9	7	8
	5				
Which ſhould be multiplied by ⅓ Part of the Height, namely, 4 F. 11 I. 11 P. 3 S. 8 T. but becauſe ⅓ Part of the Height, and ⅓ Part of the Length differ but by 8 Seconds, you may multiply it by ⅓ Part of the Length, namely, 5 Fe.	10				
	2	1			
		3	9		
			2	11	
				3	4
and the Product is	12	5	0	2	4

Which is the ſolid Content of the Segment of the Cone.

Of a Globe or Sphere.

A Globe or Sphere is a perfect round Body, contained under one Circular Plain, the middle Point whereof is called the Center, from whence all ſtrait Lines drawn to the Outſide are of equal Length, and are called Semidiameters, or rather Semiaxes.

PROP.

PROP. XIV. *Fig.* XII.

Let Fig. XII. *be a Sphere, whoſe Axis* AB *is* 8 *Feet; what is the ſolid Content thereof?*

The Rule is this:

A*S* 21 *is to* 11 (or, *As* 42 *to* 22): *So* is the Cube of the Diameter or Axis *to* the ſolid Content required.

	F.
The Axis is 8, and the Square of it	64
Which multiplied by the Axis	8
The Cube of the Axis is	512

Then ſay by the Rule of Three: If 21 give 11, what will (the Cube of the Diameter) 512 give?

Multiply 512 by 11, and the Product is 5632 (as you ſee in the Margin) which divide by the firſt Number in the Queſtion, to wit, 21, and the Quotient is 268 Feet, and $\frac{4}{21}$ of a Foot, which being reduced, is 2 Inc. 3 Parts, and 5 Seconds.

$$\begin{array}{r} 512\\ 11\\ \hline 512\\ 512\\ \hline 5632\\ \hline \end{array}$$

So that the ſolid Content of the Sphere, whoſe Axis is 8 Feet, is 268 Feet, 2 Inches, 3 Parts, and 5 Seconds.

$$\begin{array}{rrrrl} & \cancel{1} & \cancel{1} & & \\ \cancel{1} & \cancel{4} & \cancel{7} & (4 & \\ \cancel{5} & \cancel{6} & \cancel{3} & 2 & (268 \text{ Feet.}\\ \cancel{2} & \cancel{1} & \cancel{1} & \cancel{1} & \\ & \cancel{2} & \cancel{2} & & \end{array}$$

PROP.

PROP. XV.

A Sphere is given, whoſe ſolid Content is 268 *Feet,* 2 *Inches,* 3 *Parts, and* 5 *Seconds; what is the Length of the Axis?*

The Rule is this:

AS 22 *is* to 42: *So* is the ſolid Content given, to wit, 268 Feet, 2 Inches, 3 Parts, 5 Seconds, *to* the Cube of the Axis, whoſe Length is required.

State the Queſtion thus: If 22 give 42, what will 268 F. 2 I. 3 P. 5 S. give?

F.	I.	P.	S.
268	2	3	5
42			
536			
10727	10	6	
		17	6
11263	11	11	6

$\cancel{1}\cancel{2}\cancel{4}$
$\cancel{1}\cancel{1}\cancel{2}\cancel{6}\cancel{4}$ (512
$\cancel{2}\cancel{2}\cancel{2}\cancel{2}$
$\cancel{2}\cancel{2}$

Multiply the third Number by the ſecond (as in the Margin) and the Product is 11264 Feet *fere*; which divide by the firſt Number in the Queſtion 22, and the Quotient is 512 (which is the Cube of the Axis) the Cube Root whereof is 8 Feet, which is the Length of the Axis required.

The brieſeſt Way to extract Square and Cube Roots, is by a Table of Logarithms.

PROP. XVI. *Fig.* XII.

Admit in Fig. XII. CED *were a Segment (or Portion) of a Sphere, whoſe Segment of the Axis* EF *is* 2 *Feet (the whole Axis* AB *being* 8 *Feet) and the Chord (or Subtenſe) of the Segment* CD, *is* 6 *Feet,* 11 *Inches, and* 2 *Parts; what is the ſolid Content of this Portion of the Sphere?*

The Rule is this:

FIrſt, increaſe the Altitude of the other Segment (not given) by half the Axis.

Then ſay by the Rule of Three: *As* the Altitude of the other Segment not given, *is* to the Altitude (or Height) of the given Segment (or Portion) *So* is the Altitude of the other Segment increaſed by half the Axis, *to* a fourth Proportional.

Secondly, Square half the Chord (or Subtenſe) of the given Segment, and multiply the Square of that half Chord by the fourth Proportional found by the firſt Work (or Rule.)

Then ſay by the Rule of Three: *As* 21 *is* to 22: *So* is the Product of the Square of half the Chord of the Segment given, multiplied by the fourth Proportional (found as above) *to* the ſolid Content of the Segment given.

Example.

The Segment not given is CDG, whoſe Altitude FG is 6 Feet, which muſt be increaſed by half the Axis, namely, 4 Feet, and then it is 10 Feet.

Then ſay, *As* (the Altitude of the other Segment not given, which is) 6 Feet, *is* to (the Altitude of the given Segment) 2 Feet: *So* (is the Altitude of the other Segment not given, being increaſed by half the Axis, to wit, 6 Feet more 4 Feet, that is) 10 Feet, *to* a fourth Proportional required.

Let us find this fourth Proportional.

Firſt, Multiply 10 F. by 2 F. and the Product is 20 F. which divide by 6 F. and the Quotient is 3 F. $\frac{3}{4}$ Parts of a Foot, that is, 3 F. 4 I. which is the fourth Proportional.

In the next Place, Square half the given Chord CF; the whole Chord CD is 6 F. 11 I. 2 P. the half whereof is 3 F. 5 I. 7 P. *fere*, the Square whereof is 12 F. 0 I. 1 P. 2 S. + which muſt be multiplied by the fourth Proportional found, namely, 3 F. 4 I. and the Product is 40 F. 0 I. 3 P. 10 S. 8 T.

Then ſay by the Rule of Three: *As* 21 *is* to 22: *So* is the Square of half the given Chord, multiplied by the fourth Proportional found (as before) 40 F. 0 I. 3 P. 10 S. 8 T. *to* the Content required.

Then multiply the third Number by the ſecond, and the Product is 880 F. 7 I. 1 P. 6 S. 8 T. which divide by the firſt Number 21, and the Quotient is 41 F. and $\frac{19}{21}$ of a Foot, which (when reduced) is

41 F. 11 I. *fere*, which is the ſolid Content of the Segment required.

For note, That a Sphere is equal to two Cones, having the Height and the Diameter of their Baſe the ſame with the Axis of the Sphere: Or, which is all one, a Sphere is two Third Parts of a Cylinder, having the Height and the Diameter of the Baſe the ſame with the Axis of the Sphere.

But in caſe you have only the Segment of a Sphere given, and not the Axis of the whole Sphere, then you muſt find the Axis.

PROP. XVII. *Fig.* XII.

The Segment or Portion of a Sphere being given, and the Axis required;

The Rule is this:

SQuare one half of the Chord of the Segment, and the Product divide by the Altitude of the given Segment; then to the Quotient add the Altitude of the given Segment, and the Product is the Length of the Axis required.

Example.

Admit in Fig. XII. *the Segment given to be* CED, *and the Axis* AB *required to be found.*

The given Chord CD is 6 F. 11 I. 2 P. the half whereof is 3 F. 5 I. 7 P. which being ſquared, is 12 F. which divide (by the Altitude EF) 2 F. and the Quotient is 6 F. To which add the Altitude of the Segment (EF) 2 Feet, and the Product is 8 F. which is the Length of the Axis of the whole Sphere, whereof CED is a Segment.

PROP. XVIII.

The Axis of a Sphere being given; to find the ſuperficial Content:

One Rule is this:

As 7 *is* to 22: *So* is the Square of the Diameter, *to* the ſuperficial Content required.

Example.

Let the Diameter or Axis given be 8 Feet, which being ſquared is 64 Feet.

Then ſay by the Rule of Three: If 7 give 22, what will 64 give?

Multiply 64 by 22, and the Product is 1408; which divide by 7, and the Quotient is 201 Feet and $\frac{1}{7}$ of a Foot; which reduced, is 201 F. 1 I. 8 P. 7 S. *fere*, which is the ſuperficial Content of the Sphere.

On the contrary, If the ſuperficial Content of a Sphere be given, and the Axis required,

The Rule is this:

As 22 *to* 7: *So* is the ſuperficial Content given, *to* the Square of the Axis.

Example.

The Content given is 201 F. 1 I. 8 P. 7 S. what is the Axis?

Multiply the Content given by 7, and the Product is 1408, which divide by 22, and the Quotient is 64, which is the Square of the Diameter; the Square Root whereof is 8, the Axis or Diameter required.

PROP. XIX. *Fig.* XIII.

A Portion or Segment of a Sphere being given; to find the Content of the Convex Superficies.

ADmit in *Fig.* XIII. BAC to be a Portion of a Sphere, whoſe Altitude is AE 4 Feet, and whoſe Baſe or Chord is BC 8 Feet, and the Content of the Convex Superficies is required.

Draw the ſtrait Line AB from the Pole A, to the Baſe in B; then, I ſay, the Content of a Circle, whoſe Radius (or Semidiameter) is AB is equal to the Content of the Convex Superficies of the Portion BAC.

Let us try.

The Semidiameter AB is 5 F. 7 I. 10 P. 6 S. now the Content of a Circle deſcribed from that Semidiameter (being found by *Prop.* XVII. of *Meaſuring of Plains)* will be 100 F. 6 I. 6 P. which is the Content of the convex Superficies of the Portion BAC, which was required.

Which may be proved by the laſt foregoing Propoſition; for the Content Superficial of a Sphere, whoſe Diameter is 8 F. was found to be 201 F. 1 I. 8 P. +: Now this Portion B A C is half the Sphere of the ſame Axis; therefore double the Content of it, which was found to be 100 F. 6 I. 6 P. and the Product is 201 F. and 1 I. for the Content of the whole; which is the ſame that was found by the preceding Propoſition, within (or want) 8 P. which want of the 8 Parts is cauſed by the Length of the Line AB, it being a ſmall Matter more than 5 F. 7 I. 10 P. 6 S.

The Truth of this laſt Propoſition, that is to ſay, That the Circle deſcribed by the Radius A D, or AB, is equal to the convex Superficies of the Portion of a Sphere B A C, is demonſtrable from the Compariſon of Motion, thus:

Let the Plain A E B D be underſtood to make a Revolution about the Axis AE; and it is manifeſt, that by the ſtrait Line AD, a Circle may be deſcribed; and by the Arch AB, the Superficies of a Portion of a Sphere; and laſtly, by the Subtenſe AB, the Superficies of a right Cone will be deſcribed. Now ſeeing both the ſtrait Line A B, and the Arch AB, make one and the ſame Revolution, and both of them have the ſame extreme Points A and B; the Cauſe why the Spherical Superficies which is made by the Arch, is greater than the conical Superficies which is made by the Subtenſe, is, That AB the Arch is greater than AB the Subtenſe; and the Cauſe why it is greater, conſiſts in this: That although they be both drawn from A to B, yet the Subtenſe is drawn ſtrait, but the Arch angularly, namely, according to that Angle which the Arch makes with the Subtenſe, which Angle is equal to the Angle DAB (for an Angle of Contingence

tingence adds nothing to an Angle of a Segment.) Wherefore the Magnitude of the Angle DAB is the Cauſe why the Superficies of the Portion deſcribed by the Arch A B is greater than the Superficies of the right Cone deſcribed by the Subtenſe AB.

Again, The Cauſe why the Circle deſcribed by the Tangent AD, is greater than the Superficies of the right Cone deſcribed by the Subtenſe AB (notwithſtanding that the Tangent and Subtenſe are equal, and both moved round in the ſame time) is this: That AD ſtands at right Angles to the Axis, but AB obliquely; which Obliquity conſiſts in the ſame Angle D A B. Seeing therefore the Quantity of the Angle DAB is that which makes the Exceſs both of the Superficies of the Portion, and of the Circle made by the Radius A D, above the Superficies of the right Cone deſcribed by the Subtenſe AB; it follows, That both the Superficies of the Portion, and that of the Circle do equally exceed the Superficies of the Cone. Wherefore the Circle made by AD or AB, and the Spherical Superficies made by the Arch AB, are equal to one another, which was to be proved.

Of the Proportions between a Cube, a Priſm, and a Pyramid, a Cylinder, Sphere, and Cone, whoſe Altitudes and Baſes are as follow.

	F.	I.	P.	S.	T.
A *Cube*, whoſe Side or Baſe is 1 F. 4 I. the Content is	2	4	5	4	0

	F.	I.	P.	S.	T.	Fo
A *Priſm* (having its Baſe and Altitude ſeverally equal to the Baſe of the aforeſaid Cube, or of any other Rectangular Parallelepipedon is the half of it, and the Content is	1	2	2	8	0	0
A *Square Pyramid*, having its Height and Baſe ſeverally equal to the Baſe of the aforeſaid Cube, is $\frac{1}{3}$ Part of it, and its Content is	0	9	5	9	4	0
A *Triangular Pyramid*, whoſe Height is as aforeſaid, and each Side of the Baſe as afore (that is, 1 F. 4 I.) the Solid Content is	0	4	1	3	1	4
A *Cylinder*, having the ſame Height and Diameter with the Cube aforeſaid, the Solid Content of ſuch a Cylinder is	1	10	4	2	1	0
A *Sphere*, whoſe Axis is as aforeſaid, its Solid Content is	1	2	10	9	4	8
A *Cone*, whoſe Altitude and Diameter of its Baſe are ſeverally equal to the Side of the Cube aforeſaid, its Solid Content is	0	7	5	4	8	4

Hence it appears that a *Cube* is double the *Priſm*, and three times as much as the *Square Pyramid* of equal Baſe and Altitude.

The *Triangular Pyramid* is ſomething more than ½ of the Cube.

The *Cylinder*, whoſe Diameter and Height is ſeverally equal to the Height of the Cube, is in proportion to it as 11 to 14; or the Cylinder contains 11 of thoſe Parts whereof the Cube contains 14.

The *Globe* or *Sphere*, whoſe Axis is equal to the Height of a Cube, contains 11 ſuch Parts, whereof the Cube contains 21; or the Sphere is in proportion to the Cube, as 11 to 21.

The *Cone*, whoſe Diameter and Altitude are ſeverally equal to the Altitude and Diameter of the Cylinder, is in proportion to the Cylinder as 1 to 3.

Whence it appears, that the Sphere is ⅔ of the Cylinder, and the Cone ½ of the Sphere.

Of GAUGING.

Gauging is comprehended in the Meaſuring of *Solids*, and there is only this Difference between Gauging (or meaſuring of Veſſels) and meaſuring of other Solids: The Content of the latter is given in Feet, Inches, Parts, *&c.* but the Content of the former is given in Gallons, Quarts, Pints, *&c.*

But before you can give the Content in Gallons, *&c.* you muſt firſt find the ſolid Content in Cubick Inches, Parts, *&c.* and having firſt found the Content in Inches and Parts, you muſt afterwards reduce them into Gallons, Quarts, and Pints, *&c.*

There are ſeveral Methods ſhewn for the finding the Content of theſe irregular Solids; but the Method ſhewn by Mr. *Oughtred*, is generally eſteemed for one of the beſt; and therefore I ſhall make uſe of his Method.

Moſt Liquid Veſſels, ſuch as are Pipes, Hogſheads, Barrels, Kilderkins, Firkins, *&c.* are made in form of a Spheroides, having the two Ends equally cut off; and accordingly may be meaſured thus:

Meaſure the two Diameters of the Veſſel in Inches, the one at the Bung-hole, the other at the Head, and alſo the length within; and by the Diameters found, find out the Areas, or Contents of the Circles: Then add together two third Parts of the Content of the greater Circle, and one third Part of the Content of the leſſer Circle. Laſtly, Multiply the Product of theſe two Sums added together, by the Length of the Veſſel; ſo ſhall you have the Content of the Veſſel in Cubick Inches.

Of which 231 make a Wine Gallon; and 272, and 3 Parts, make an Ale or Beer Gallon, according to Mr. *Oughtred*, who would have a Gallon to conſiſt of a Number of Cubick Inches, the Square Root whereof is Palms 5 ½.

That 231 Cubick Inches make a Wine Gallon, is the Opinion of moſt Men; but the Quantity of the Ale or Beer Gallon, is not as yet fully agreed on: Formerly the Ale Gallon hath been accounted to contain 288 Cubick Inches, and ¾ or 9 Parts; but ſince the Exciſe, it is computed to contain but 282 Cubick Inches.

PROP. XX.

Admit a Veſſel to have the Diameter at the Bung 32 *Inches, and at the Head* 18 *Inches, and the Length* 40 *Inches, what is the Content of this Veſſel in Gallons, and Parts of a Gallon?*

THeſe Dimenſions being given in Inches, and the Content required in Inches, you muſt have reſpect to the Note at *Page* 288. Book 5.

We muſt, firſt of all, find the Content of the two Circles belonging to the two Diameters given, by the ſecond Rule delivered at the 17th Propoſition of the Fifth Book.

	Inches.
The Square of the Diameter 32, is	1024
Which according to the Rule, multiply by	355

```
      5120
     5120
    3072
    ------
And the Product is   363520
```

Which divide by 452 (according as the aforeſaid Rule directs) and the Quotient is (as in the Margin) 804 Inches, and $\frac{112}{452}$ Parts of an Inch; which reduced, is 804 I. 2 P. 11 S. 8 T. for the

```
   (1
   19(12    I.
 363520 (804
  45222
   455
    4
```

Content of the greater Circle, to wit, the Circle at the Bung, whoſe Diameter was 32 Inches.

	Inches.
In the next Place, the Square of the Diameter 18 Inches, is	324
Which multiply by	355
	1620
	1620
	972
And the Product is	115020

Which divide by 452 (as in the Margin) and the Quotient is 254 Inches, and $\frac{212}{452}$ of an Inch; which reduced, is 254 I. 5 P. 7 S. 6 T. and is the Content of the leſſer Circle at the Head, whoſe Diameter was 18 Inches.

```
     (2
    20(1
   2462(2
 115020 (254Inches.
  45222
   455
    4
```

The next Work is, to take the two third Parts of the Content of the greater Circle, and one third Part of the Content of the leſſer Circle, and add them together.

	I.	P.	S.	T.
The Content of the greater Circle, is	804	2	11	8
The Content of the leſſer Circle, is	254	5	7	6
Two third Parts of the greater Circle, is	536	1	11	9 +
And one third Part of the leſſer Circle, is	84	9	10	6
Which being added, the Product is	620	11	10	3
Which multiply by the Length	40			
	24800	33	4	
	36	8	10	
The Content of the Veſſel in Cubick Inches, is	24839	6	2	

The next Work will be, to find how many Gallons is contained in 24839 Cubick Inches.

And if you would know the Content in Wine Meaſure, divide the Number of Inches by 231, and the Quotient gives you the Content in Wine Gallons.

But if you would know the Content in Ale Meaſure, then divide the Number of Cubick Inches by 282, and the Quotient gives you the Content: Or if you would know the Content in Ale Meaſure, according as the Gallon was accounted (before the Exciſe) to contain 288 Cubick Inches, and ¾ (or 9 Parts.) After you have reduced the Inches into Wine Meaſure, then multiply the Content in Gallons (and Parts, if there be any) by 4, and divide the Product by 5, and the Quotient gives you the Content required; for 231 being multiplied by 5, produces 1155 I. which being divided by 4, the Quotient is 288 I. and $\frac{3}{4}$ (or 9 P.) for 231 bears the ſame Proportion to 288 Inches and 9 Parts, that 4 does to 5.

Example.

Suppoſe you would know how many Gallons of Wine Meaſure is contained in 24839 Cubick Inches; divide the Number given by 231 (as in the Margin) and the Quotient is 107 Gallons, and $\frac{122}{231}$ Parts of a Gallon; which reduced, is 107 Gallons and an half, and about $\frac{1}{4}$ of a Pint.

$(1$
$\cancel{1}\cancel{7}(22$
$\cancel{2}\cancel{4}\cancel{8}\cancel{3}\cancel{9}$ (107 Gallons.
$\cancel{2}\cancel{3}\cancel{1}\cancel{1}\cancel{1}$
$\cancel{2}\cancel{3}\cancel{3}$
$\cancel{2}$

Then if you would know how many Ale Gallons it contains, accounting 288 I. 9 P. to the Gallon, ſay, *As* 5 *is* to 4; *ſo* is the Content in Wine Gallons, *to* the Content in Ale Gallons.

Multiply 107 Gallons and an half, by 4, and the Product is 430, which divide by 5, and the Quotient is 86 Gallons, and near a Quarter of a Pint.

$107\frac{1}{2}$
4
———
428
2
———
430
———

$\cancel{3}$
$\cancel{4}\cancel{3}\cancel{0}$ (86 Gal. $\frac{1}{4}$ Pint.
$\cancel{5}\cancel{5}$

Or if you would know how many Ale Gallons (of 288 I. 9 P. to the Gallon) there is in 24839 I. 6 P. without reducing it firſt to Wine Meaſure, do thus:

Multiply the Content given in Inches, by 12 (to bring them into Parts) and the Product is 298068, to which add the 6 Parts that are in the Content beſides the Inches, and the Product is 298074 P. (as in the Margin.)

I. P.
24839 6
12 0
49678
24839
298068
6
298074

Then reduce the Inches of the Gallon into Parts, by multiplying 288 by 12, and the Product is 3456; to which add the 9 Parts, and the Product is 3465 Parts; laſtly, divide 298074 by 3465, and the Quotient is 86 Gallons, and $\frac{84}{3465}$ of a Gallon; which 84 Parts being reduced is 7 Inches, which is not a quarter of a Pint, for a quarter of a Pint contains 9 Inches.

I. P.
288 9
12 0
576
288
3456
9
3465

By the ſame Method you may find the Content (of any Number of Inches and Parts given) in Ale Meaſure, allowing (with M. *Oughtred*) 272 Cubick Inches and 3 Parts to the Gallon.

(8
2087 (4
298074 (86 Gallons.
34655
346

Now becauſe the finding the two third Parts of the Content of the Circle at the Bung, and the one third Part of the Content of the Circle at the Head, is ſomething tedious; I have therefore found out proportional Numbers to ſhorten the Work, which are as follow.

As 1728 I. *is* to 904 I. 9 P. 4 S. 5 T. 2 Fo. *ſo* is the Square of the Diameter at the Bung, *to* two third Parts of the Content of the Circle thereof.

Again: *As* 1728 I. *is* to 452 I. 4 P. 8 S. 2 T. 7 F. *ſo* is the Square of the Diameter at the Head, *to* one third Part of the Content of the Circle thereof.

Having theſe proportional Numbers, the Operation will be after this manner.

Firſt, Square each Diameter.

Secondly, Multiply the Square of the Diameter at the Bung by 904 I. 9 P. 4 S. 5 T. 2 Fo.

Thirdly, Multiply the Square of the Diameter at the Head by 452 I. 4 P. 8 S. 2 T. 7 Fo.

Fourthly, Add theſe two Products together.

Fifthly, Divide the Product of the Addition by 1728.

Sixthly, Multiply the Quotient by the Length of the Veſſel, and the Product is the Content in Cubick Inches.

Example.

	I.	P.	S.	T.	Fo.
I. The Diameter at the Bung is 32 Inches, the Square whereof is	1024	0	0	0	0
II. Which multiply by	904	9	4	5	2
	4096	341	4		
	92160		426	8	
	768			170	8
The Product is	926495	6	0	10	8

III. The

	I.	P.	S.	T.	Fo.
III. The Diameter at the Head is 18 Inches, the Square whereof is	324	0	0	0	0
Which multiply by	452	4	8	2	7
	648	216	54		
	1620			189	
	129608				
	1				
The Product is	146574	5	9	9	0
IV. To which add the former Product	926495	6	0	10	8
And the Sum is	1073069	11	10	7	8

V. Divide this laſt Sum by 1728, and the Quotient is 620 I. and $\frac{1709}{1728}$ Parts of an Inch, which being reduced, is 620 I. 11 P. 10 S. 5 T.

```
     (17
    362(0      I.
  107306(9 (620
   172888
    1722
     17
```

VI. Multiply this laſt Sum by the Length of the Veſſel 40 I. and the Product is (as in the Margin) 24839 I. 6 P. 8 S. 8 T. which is the ſolid Content in Cubick Inches and Parts, and the ſame that was found the firſt Way.

I.	P.	S.	T.
620	11	10	5
40			
24800	33	4	
36	8	16	8
24839	6	8	8

For meaſuring or gauging of *Brewers* Tuns or Veſſels, whether they be ſquare or round, or any other Form, work thus:

Firſt find the Content in Cubick Inches, by the preceding Rules of meaſuring ſuch Bodies; then divide that Content by 282 (the Number of Inches agreed upon for a Gallon to contain ſince the Exciſe) and the Quotient is the Content in Gallons. Laſtly, to bring the Gallons into Barrels, divide the Number of them by 36 (the Gallons in one Barrel) and the Quotient gives you the Content of the Veſſel or Tun, in Barrels.

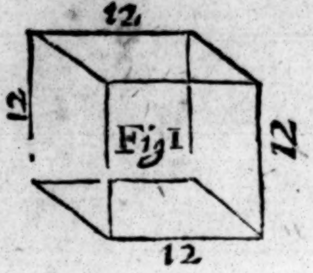


Fig I

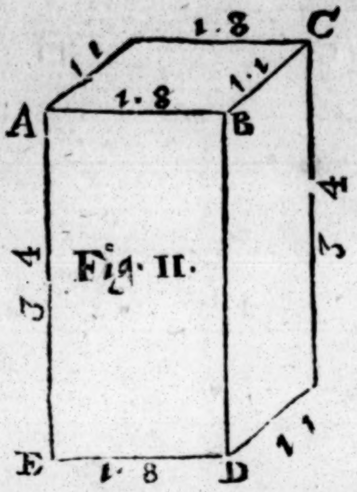


Fig. II.

Fig.VIII

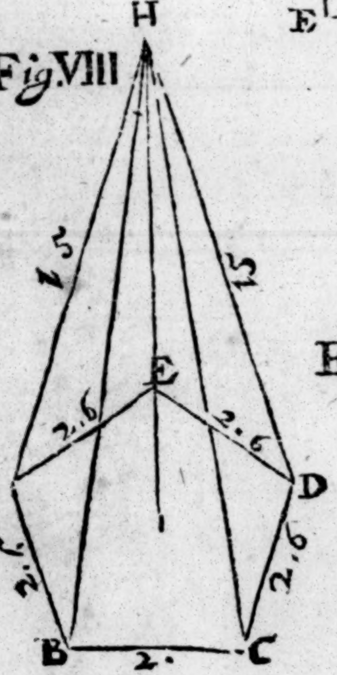


Fig.IX

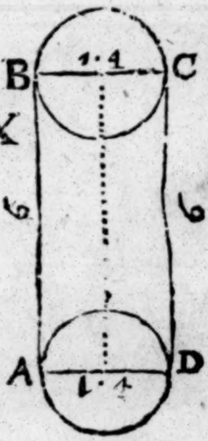


Fig X

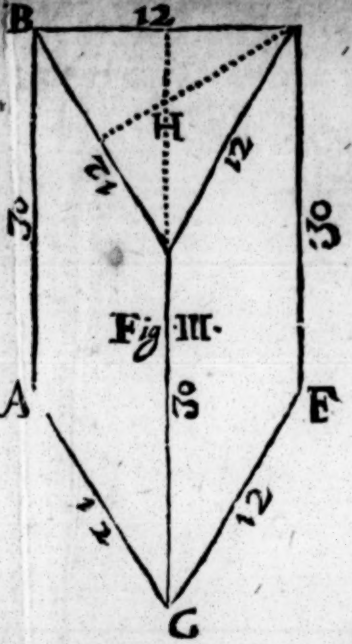


Fig III·

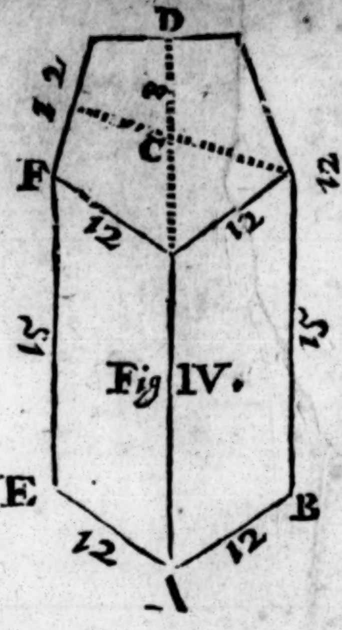
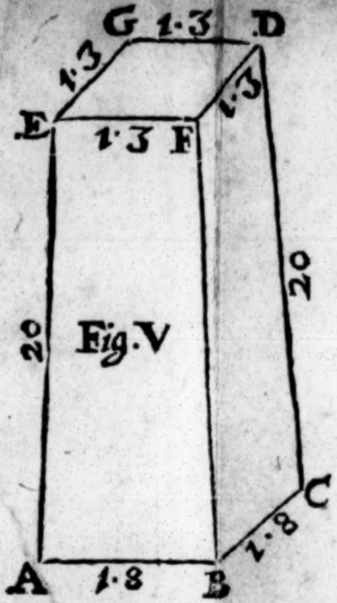
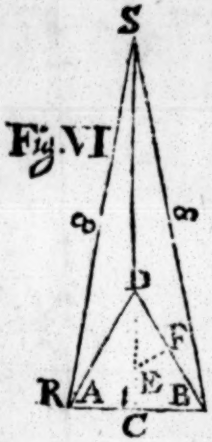


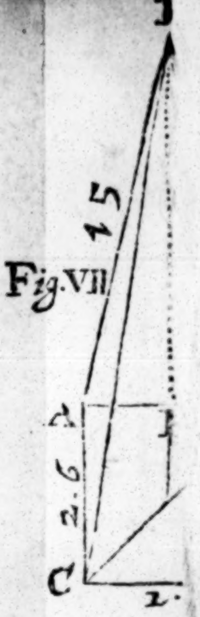
Fig IV·



Fig·V



Fig·VI



Fig·VII

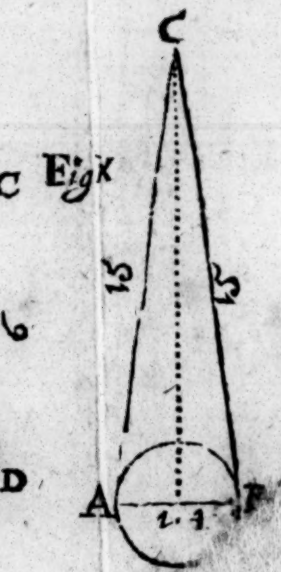


Fig X

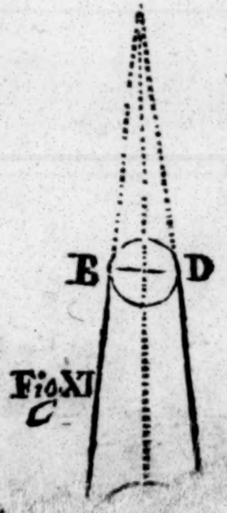
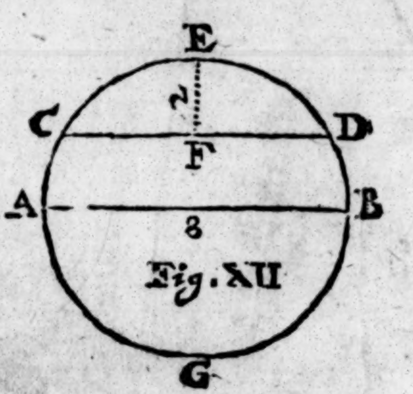
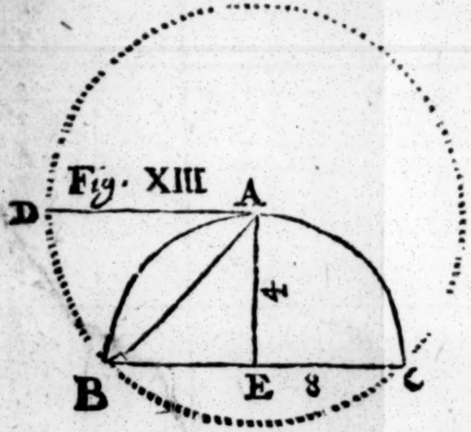


Fig XI



Fig·XII



Fig·XIII

his at the End of the Sixth Book.

THE
APPENDIX.

Treating in the FIRST PLACE
OF THE
MEASURING
OF
CHIMNEYS
REFORMED.

By *VEN. MANDEY.*

LONDON:
Printed in the YEAR M.DCC.XXVII.

THE

APPENDIX.

Treating in the firſt place of the Meaſuring of Chimneys Reformed.

THE Brick-work of Chimneys being a great part of it not to be ſeen, hath occaſioned much Error in the Meaſuring thereof; and, it may be, not one Meaſurer in ten is ſenſible how the Funnels are wrought, and in what manner they are carried up: I thought it might be a Service to the Publick to correct this Error, that ſo neither the Gentleman, nor he that pays for the Work, nor the Workman that doth it, may receive any Injury. The uſual way of girting round the Jaumbs and Breaſt of the Chimney, for the Length to be multiplied by the Height of the Story, is erroneous, and more in one Story than in another, as will appear hereafter. Some Years ago, it was my Hap to meaſure the Bricklayers Work of an Houſe, wherein Mr. *Leonard Sowersby*, the late notorious Meaſurer, was concerned againſt me. We had oftentimes meaſured together before that time, and alſo ſince in ſeveral

Places: But here we happen'd to diſagree about meaſuring the Chimneys very conſiderably, which gave me occaſion to delineate the Chimneys on Paper, and to ſhew him wherein he was miſtaken: The Chimneys were a double Stack placed Diagonal ways againſt the Party Wall, as you may ſee in Fig. 1. But before we proceed any farther, it will be convenient that you underſtand ſome Symbols which I ſhall have occaſion to uſe in demonſtrating the Menſuration of the following Particulars in this Appendix, only to abbreviate the Work.

Symbols.	*Signification.*
=	Equal to.
+	More.
—	Leſs.
×	Multiplied by
dd	Deduct.
△	Triangle.
□	Square.
▭	Oblong.
Z	The Sum.
X	The Difference.
1 B	One Brick-work.
1½B	One Brick and half work.
f. i.	Feet. Inches.

Proceed we now to the Meaſure of theſe two Chimneys in the Shop or Parlour Story; for in the Cellar there were Piers wrought, and Arches turn'd, if my Memory fail not.

And firſt, Mr. *Sowerſby*'s way was thus: He girt them round from the Wall, which Length was 16 *f.* 2 *i.* and multiplied that by the Height of the Story, which was 10 *f.* 8 *i.* at 1 Brick in Thickneſs, and would allow no more.

feet	*in.*	
16	2	
10	8	
160	0	
1	8	
10	8	
	1	
172	5	of 1 Brick-work, which being reduced, is 114 *f.* 11 *i.* of one Brick and ½.

I told him, that he had not allowed the true Meaſure of the Work by a great deal, and that I did conceive the trueſt and readieſt way, was to meaſure them as a Solid, and deduct the Vacancies; which accordingly I did in the following manner: The two Chimneys made the half of a Quadrate, each ſide being 8 *f.* 1 *i.* which Quadrate was divided in two in the middle by the Party Wall, as you may perceive by Fig. 1. Therefore I multiply 8 *f.* 1 *i.* by 8 *f.* 1 *i.* it makes 65 *f.* 4 *i.* half whereof is 32 *f.* 8 *i.* which being multiplied by the Height of the Story, *viz.* by 10 *f.* 8 *i.* is = 348 *f.* 5 *i.* the ſolid Content: In the next place we muſt deduct the Vacancies; the Chimneys were both alike in Magnitude, and each Chimney was 4 *f.* wide between Jaumb and Jaumb, and 1 *f.* 8 *i.* deep, from Breaſt to Back, and the Mantle-trees lay 4 *f.* high from the Floor. The neareſt way, as I conceive, is to deduct the whole as a Vacancy, and afterwards to add the Breaſts, Wings, and Wieths: The Vacancy of one Chimney is 4 *f.* × 1 *f.* 8 *i.* = 6 *f.* 8 *i.* × 10 *f.* 8 *i.* = 71 *f.* 1 *i.* which being doubled for both Chimneys is 142 *f.* 2 *i.* which deducted from the Solidity, leaves Cubick feet, 206 *f.* 3 *i.* which muſt be reduced to 1 Brick and half in thickneſs, by ſaying, As 14 to 12, ſo 206 *f.* to 176 *f.* 6 *i.* of 1 Brick and half Work. Now becauſe we have deducted all the Breadth and Depth of the Chimneys, by the Height of the Story, and there being the Breaſts, and Wings, and Wieths, included in this Deduction, we muſt add the Quantity of them to the 176 *f.* 6 *i.*

The mean Length of one Wing, and both the four Inches of the Breaſt, and the croſs Wieth, is 8 *f.* 5 *i.* which multiplied by 6 *f.* 8 *i.* the Height of the Mantle-tree to the Top of the Story, is = 56 *f.* 1 *i.* of ½ Brick-work, which reduced is 18 *f.* 8 *i.* of one Brick and half work in one Chimney, the double of which for both Chimneys, is 37 *f.* 4 *i.* which added to 176 *f.* 6 *i.* makes 213 *f.* 10 *i.* of one Brick and half work, the true Content of theſe two Chimneys in the Parlour Story: Then we muſt add the Pargetting of theſe two Funnels in this Story, (for ſince we meaſure no more Brick-work, than is really done in the Chimneys, we ought in Juſtice to meaſure, and allow for the Pargetting; by reaſon the Chimneys are more troubleſome in working than ſtrait Walls; and yet when they are meaſured by the Rod, the Bricklayer hath no greater Price by the Rod for them, than he hath for working the ſtrait Walls: The mean Girt of the four Sides of one Funnel, is 5 *f.* 8 *i.* which multiplied by 6 *f.* 8 *i.* = 37 *f.* 9 *i.* which doubled for both Funnels, is 75 *f.*

6 *i.* which divided by 9 is 8 Yards + 3 *f.* + 6 *i.* of Pargetting, at 4 *d. per* Yard, is 2 *s.* 9 *d.*

	feet.	*in.*	
So that the true Meaſure of theſe two Chimneys, is	213	10	of 1 ½ Brick-work.
And Mr. *Sowersby* made the Content to be but	114	11	
The Difference	98	11	

A very conſiderable Quantity of Work, for the Workmen to loſe in one Story, beſides the Pargetting of the Funnels, and ſo proportionally in every Story.

EXAMPLE II.

Suppoſe the Dining-Room Story to be 11 *f. high, and the Chimneys the ſame Girt as in the Parlour; See Fig.* 1. *and the ſame Bigneſs within.*

Firſt, Mr. *Sowersby*'s Way was,

feet.	*in.*	
16	2	
11	0	
16		
161	10	
177	10	of 1 Brick in Thickneſs, being = to
118	6	of 1 Brick ½ Work.

Now my way,

	feet.	in.
The Solidity	8	1
	8	1
	64	8
	8	8
	65	4
½ =	32	8
which ×	11	0
	32	
	327	4
The Solid Content	359	4

The Solid Content of the two Chimneys in the Dining-Room Story, is 359 *f.* 4 *i.* of Cubick Feet; now we muſt deduct the two Funnels of the Parlour Story, and the two Funnels of this Story: One Funnel of the Parlour Story, is 1 *f.* 2 *i.* × 1 *f.* 2 *i.* = 1 *f.* 4 *i.* × 11 *f.* the Height of the Story is = 14 *f.* 8 *i.* the Double whereof, for both the Parlour Funnels, is 29 *f.* 4 *i.* Then ſuppoſing the Chimneys in the Dining-room Story to be the ſame Breadth between the Jaumbs, and the ſame Depth as in the Parlour Story, we will deduct the Breadth by the Depth, the whole Height of the Story, and add the Wings, and Breaſts, and Wieths afterwards: Well then 4 *f* × 1 *f.* 8 *i.* = 6 *f.* 8 *i.* × 11 *f.* the Height of the Story, is = 73 *f.* 4 *i.* for one Chimney, the double whereof for both Chimneys, is 146 *f.* 8 *i.* which added to 29 *f.* 4 *i.* the other Deduction makes 176 *f.* which muſt be deducted from the Solidity, and there remains Cubick Feet 183 and 4 *i.* Say then, if 14 give 12, what will 183 give? Multiply and divide according to the Rule of Proportion, and you will have 156 *f.* 10 *i.* of one Brick and half Work, to which we muſt add the Wings, and Breaſts, and Wieths: The mean Length of both Wings, and of the half Brick-work of the inſide and outſide Breaſt, and croſs Wieth, is 10 *f.* 5 *i.* for one Chimney, the double whereof, is 20 *f.* 10 *i.* for both Chimneys, which multiplied by 7 *f.* the Height of them, from the Mantle-tree to the Top of the Story, produces 145 *f.* 10 *i.* of half Brick-work, in the Breaſts, *&c.* of both Chimneys, which reduced, is 48 *f.* 7 *i.* of one Brick and half Work, to be added to 156 *f.* 10 *i.* which makes 205 *f.* 5 *i.* of 1 Brick and half Work in the Dining-Room Story, being 86 *f.* 11 *i.* more than Mr. *Sowerſby*'s way: next we muſt meaſure the Pargetting of the four Funnels: The four Sides of one of the Parlour Funnels, is = 4 *f.* 8 *i.* (for we ſuppoſe the Funnels to be 14 *i.* □) which multiplied by 11 *f.* the Height of the Story, is = 51 *f.* 4 *i.* the Double whereof is 102 *f.* 8 *i.* for both Funnels: And for the Pargetting of the other two Funnels, from the Mantle-tree, to the Top of the Story, the mean Length of the 4 Sides, is 5 *f.* 8 *i.* × 7 *f.* the Height from the Mantle-tree to the Top of the Story, is = 39 *f.* 8 *i.* for one, the Double whereof, is 79 *f.* 4 *i.* for both Funnels, which added to 102 *f.* 8 *i.* the Pargetting of the Parlour Funnels makes 182, which divided by 9, produces 20 Yards and 2 *f.* which at 4 *d.* *per* Yard, comes to 6 *s.* + 9 *d.*

EXAMPLE III.

Of Meaſuring Chimneys that ſtand at right Angles, adjoining to a Party or other Wall, as Fig. 2.

Suppoſe Fig. 2. to be the firſt Story: The uſual way is to girt the Chimney about, for the Length, and multiply that Length by the Height of the Story, allowing the Work to be of the ſame Thickneſs as the Jaumbs, which in ſome Caſes it comes pretty near the Truth, and in others it makes leſs than the true Quantity by a great deal: The Jaumbs of this Chimney being 3 *f.* from the Wall, and the Chimney 4 *f.* within, and the Jaumbs 9 *i.* in Thickneſs, the Girt is 11 *f.* 6 *i.* × by 10 *f.* the Height of the Story, is = 115 *f.* of one Brick-work, which reduced is = 76 *f.* 8 *i.* of one half Brick-work. Let's try it by meaſuring each Particular; the two Jaumbs being added together, make 6 *f.* × 10 *f.* = 60 *f.* the Back, is 4 *f.* × 10 *f.* = 40 *f.* of two Brick-work, which being doubled is = 80 *f.* of one Brick-work: The outſide Breaſt is 4 *f.* × 6 *f.* (that is from the Vacancy under the Mantle-tree, to the Top of the Story, ſuppoſing the Mantle-tree to lie 4 *f.* high from the Floor,) is = 24 *f.* of half Brick-work, = 12 *f.* of one Brick-work: Then the inſide Breaſt, and Wing, and Wieth, the mean Length, is 3 *f.* 9 *i.* × 6 *f.* = 22 *f.* 6 *i.* of half Brick-work, = 11 *f.* 3 *i.* of one Brick-work. All theſe being added together, makes 163 *f.* 3 *i.* of 1 Brick-work, being = 108 *f.* 10 *i.* of one half Brick-work, out of which we muſt deduct the falling back for the Funnel, being 1 *f.* 2 *i.* × 7 *f.* = 8 *f.* 2 *i.* of one half Brick-work, and there remains

100 *f.* 8 *i.* one and a half Brick-work, being 24 *f.* more than the uſual way. But if the Jaumbs had been one Brick and a half in Thickneſs, the uſual way of Meaſuring would have wanted but 5 *f.* 8 *i.* of the true Meaſure or Quantity; the Pargetting muſt be added, which is about 4 Yards, at 4 *d.* *per* Yard, comes to 1 *s.* 4 *d.*

EXAMPLE IV.

To meaſure Fig. 2. *in the ſecond Story.*

Suppoſing it to be 11 *f.* high to the Top of the Floor, and the Girt the ſame, as in the firſt Story: 11 *f.* 6 *i.* × 11 *f.* = 126 *f.* of one Brick-work = 84 *f.* 4 *i.* of one and a half Brick-work, according to the uſual way of meaſuring: We will try it the true way, the two Jaumbs = 6 *f.* × 11 *f.* = 66 *f.* of one Brick-work. The Back 4 *f.* × 11 *f.* = 44 *f.* of two Brick-work = 88 *f.* of one Brick-work; the outer Breaſt, 4 *f.* × 7 *f.* = 28 *f.* of half Brick-work = 14 *f.* of one Brick-work; the inner Breaſt, and Wing, and Wieth, is 3 *f.* 9 *i.* the mean Length, which × 7 *f.* the Height from the Mantle-tree to the Top of the Story, is = 26 *f.* 3 *i.* of half Brick-work = 13 *f.* 1 *i.* of one Brick-work, all being added, makes 181 *f.* 1 *i.* of one Brick-work, which reduced, makes 120 *f.* 8 *i.* of one and a half Brick-work, out of which we muſt deduct the Funnel of the firſt Story, and the Funnel of this Story: The Funnel of the firſt Story, is 1 *f.* 2 *i.* × 11 *f.* = 12 *f.* 10 *i.* of one and a half Brick-work, (for we ſuppoſe the Funnels to be 14 *i.* □ within) the Funnel of this Story, is 1 *f.* 2 *i.* × 7 *f.* = 8 *f.* 2 *i.* of one and a half Brick-work, which added to the 12 *f.* 10 *i.* makes 21 *f.* to be deducted from 120 *f.* 8 *i.* ſo there remains 99 *f.* 8 *i.* of one and a half Brick-work, being 15 *f.* 4 *i.* more than the uſual way. Then alſo the Pargetting of theſe two Funnels is to be meaſur'd; the four Sides of the firſt Story Funnel being added together, is = 4 *f.* 8 *i.* × 11 *f.* = 51 *f.* 4 *i.* the Sides of the Funnel in this Story being added, the mean Length is 5 *f.* 4 *i.* × 7 *f.* = 37 *f.* 4 *i.* both being added, produce 88 *f.* 8 *i.* which is nine Yards; and 7 *f.* of Pargetting, at 4 *d.* *per* Yard, comes to 3 *s.* 3 *d.*

EXAMPLE V.

To meaſure Fig. 2. *in the third Story.*

Suppoſing the Story to be 9 *f.* 6 *i.* to the Top of the Floor, and the Girt the ſame as in the two other Stories, 11 *f.* 6 *i.* × 9 *f.* 6 *i.* = 109 *f.* 3 *i.* of one Brick-work, = 72 *f.* 10 *i.* of one and a half Brick-work according to the uſual way. Now for the true way, the two Jaumbs added make 6 *f.* × 9 *f.* 6 *i.* = 57 *f.* of one Brick-work. The Back 4 *f.* × 9 *f.* 6 *i.* = 38 *f.* of half Brick-work = 19 *f.* of one Brick-work; the two Wieths of the Funnels, is 2 *f.* 4 *i.* × 9 *f.* 6 *i.* = 22 *f.* 2 *i.* of half Brick-work = 11 *f.* 1 *i.* of one Brick-work; the outſide Breaſt 4 *f.* × 5 *f.* 6 *i.* = 22 *f.* of half Brick-work, 11 of one Brick; the two Wings and inſide Breaſt, and Wieth, the mean Length, is 5 *f.* × 5 *f.* 6 *i.* = 27 *f.* 6 *i.* of half Brick-work = 13 *f.* 9 *i.* of one Brick-work; all theſe being added, make 111 *f.* 10 *i.* of one Brick-work = 74 *f.* 7 *i.* of one and a half Brick-work, which exceeds the uſual way, but 1 *f.* 9 *i.* beſides the Pargetting: For which, the four Sides of one Funnel, is 4 *f.* 8 *i.* × 9 *f.* 6 *i.* = 44 *f.*

4 *i.* the other Funnel being the ſame, is = 44 *f.* 4 *i.* and the Funnel in this Story, the mean Length of the four Sides, is 8 *f.* 2 *i.* × 5 *f.* 6 *i.* = 44 *f.* 11 *i.* all three being added, make 133 *f.* 7 *i.* = 14 Yards and 7 *f.* at 4 *d. per* Yard, comes to 4 *s.* 11 *d.* which added to the Price of the Exceſs of the Brick-work, is 5 *s.* 6 *d.* being too much for a Workman to loſe in one Story.

EXAMPLE VI.

Of Meaſuring Angle Chimneys, or Chimneys ſtanding in an Angle.

We will ſuppoſe Fig. 3. to be the Chimney in the Cellar, or Ground Story, and to be 9 *f.* high from the Foundation to the Top of the Floor. The uſual way of Meaſuring this Chimney, is to take the Length of the Foreſide, or Breaſt, and to multiply it by the Height of the Story, and to allow the Work to be half Brick thinner than the Breadth of the Jaumbs; nay ſome will allow the Work to be but one Brick thick, let the Jaumbs be of never ſo much in Thickneſs: Well then the Length of the Foreſide of Fig. 3. is 7 *f.* × 9 *f.* = 63 *f.* of one and a half Brick-work, becauſe the Jaumbs are two Bricks thick: We will try it the true way; ſuppoſing the Walls A and B that are at right Angles to be old Walls, or if new, to be meaſured before by themſelves: But before we proceed to the true way of Meaſuring theſe kind of Chimneys, it will be convenient to inſert a Rule, whereby we may know by Meaſuring the Foreſide, what the Length of the two ſide Walls A and B are, which are included by the Foreſide, which is from the 47 Propoſition of the firſt Book of *Euclid*; and 'tis thus, Square the Length of the Foreſide, and take ¼ part of the Product for the ſuperficial Content: This Rule is of good uſe for the deducting of any Angle Chimney out of Cieling, or Paving, or Flooring, &c. Therefore note it well, for it is of great uſe. Now to come to the Buſineſs, the Length of the Foreſide is 7 *f.* which ſquared, or × 7 *f.* = 49 *f.* a fourth Part of it is 12 *f.* 3 *i.* for the ſuperficial Content, which multiplied by the Height of the Story, is 9 *f.* produces Cubick Feet 110 *f.* 3 *i.* then we will deduct the Vacancy of the whole Length, Breadth, and Height, including the Breaſts, and Wings, and add them afterwards; well then 4 *f.* × 1 *f.* 6 *i.* = 6 *f.* × 9 *f.* = 54 *f.* which deducted from 110 *f.* 3 *i.* leaves Cubick Feet 56 *f.* 3 *i.* which muſt be reduced to one and a half Brick-work, by making it as 14 to 12, ſo 56 *f.* 3 *i.* to 48 *f.* 3 *i.* of one and a half Brick-work. In the next place, we muſt deduct the Funnel from the Mantle-tree upwards, which muſt ſtand in the Triangle, let it be 1 *f.* 2 *i.* × 1 *f.* = 1 *f.* 2 *i.* × 5 *f.* = 5 *f.* 10 *i.* of one and a half Brick-work, which deduct from 48 *f.* 3 *i.* and there remains 42 *f.* 5 *i.* of one and a half Brick-work: In the next place we muſt add the Breaſts, and Wings, and Wieths: The outſide Breaſt, is 4 *f.* × 9 *f.* = 36 *f.* of half Brick-work = 12 *f.* of one and a half Brick-work: The inſide Breaſt, and Wing, and Wieth, is 5 *f.* × 5. *f.* (the Height from the Tarſels to the Top of the Floor) makes 25 *f.* of half Brick-work = 8 *f.* 4 *i.* of one and a half Brick-work; theſe two being added, make 20 *f.* 4 *i.* of one and a half Brick-work, which added to the 48 *f.* 3 *i.* we had before, makes 68 *f.* 7 *i.* of one Brick and a half Work, for the true Content of this Chimney, being 5 *f.* 7 *i.* more than the uſual way makes it, the Pargetting is likewiſe to be meaſured, and allowed for.

E X-

EXAMPLE VII.

Suppoſing the ſame Chimney in Fig. 3. to be 4 *f.* wide, and the Jaumbs but 14 *i.* thick on the Foreſide in the ſecond Story, which Story we will for Example ſuppoſe to be 10 *f.* high; the Length of the Foreſide will be 4 *f.* + 1 *f.* 2 *i.* + 1 *f.* 2 *i.* = 6 *f.* 4 *i.* which multiplied by 10 *f.* makes 63 *f.* 4 *i.* of one Brick-work, if you allow the Work to be half a Brick thinner than the Jaumbs, as moſt Meaſurers generally do, which reduced, makes 42 *f.* 2 *i.* of one and a half Brick-work. But to find the true Quantity of the Chimney in this Story, we work thus; the Length of the Foreſide is 6 *f.* 4 *i.* which, when ſquared, makes 40 *f.* 1 *i.* 4 *p.* a fourth Part of it is 10 *f.* (rejecting the 1 *i.* and 4 *p.*) for the Area of the Baſe, which multiplied by 10 *f.* the Height of the Story produces 100 Cubick Feet: Then we muſt deduct the Vacancy of the Funnel in the firſt Story, being 1 *f.* 2 *i.* × 1 *f.* × 10 *f.* = 11 *f.* 8 *i.* Alſo we muſt deduct the Vacancy in this Story, being 4 *f.* × 1 *f.* 6 *i.* = 6 *f.* × 10 *f.* = 60 *f.* which added to the other Deduction 11 *f.* 8 *i.* makes 77 *f.* 8 *i.* which taken from 100 *f.* leaves Cubick Feet 22 *f.* 4 *i.* which muſt be reduced to one and a half Brick-work, by making it as 14 to 12, ſo 22 *f.* 4 *i.* to 19 *f.* of one and a half Brick-work, to which we muſt add the Breaſts, and Wieths, and Wings, the mean Length of which, is 9 *f.* × 6 *f.* = 54 *f.* of half Brick-work = 18 *f.* of one and a half Brick-work, which added to the 19 *f.* of one and a half Brick-work, makes 37 *f.* of one and a half Brick-work; to which the Pargetting muſt be added of both Funnels.

EXAM-

EXAMPLE VIII. Fig. 4.

Suppoſing Fig. 4. to be a double Stack of Chimneys in the Ground Story ſtanding ſingly by themſelves, and the Height of the Story, to be 10 *f.* the Chimneys to be each 4 *f.* wide, and 18 *i.* deep, and the Jaumbs one Brick thick: To meaſure theſe Chimneys the uſual Way, is to girt them about, or to take the Length of one Jaumb, and the Foreſide, and double them for the Length, and multiply it by the Height of the Story; the Length of one Jaumb is 5 *f.* and the Length of the Foreſide is 5 *f.* 6 *i.* which being added, makes 10 *f.* 6 *i.* the Double whereof is 21 *f.* for the Girt, or Length, which multiplied by 10 *f.* the Height of the Story is = 210 *f.* of one Brick-work, which reduced, is = 140 *f.* of one and a half Brick-work, according to the uſual way: Let's try it the true way; the two Jaumbs added, are 10 *f.* which × 10 *f.* = 100 *f.* of one Brick-work = 66 *f.* 8 *i.* of one and a half Brick-work: The Back 4 *f.* × 10 *f.* = 40 *f.* of two and a half Brick-work = 66 *f.* 8 *i.* of one and a half Brick-work: One outſide Breaſt being 4 *f.* in Length, the other being the ſame, and added makes 8 *f.* of half Brick-work; the mean Length of the two inſide Breaſts, and Wings, and Wieths, is 6 *f.* 9 *i.* which add to the 8 *f.* and it makes 14 *f.* 9 *i.* which × by 6 *f.* the Height from the Tarſels to the Top of the Story, makes 88 *f.* 6 *i.* of half Brick-work, which when reduced, produces 59 *f.* of one and a half Brick-work, which three Contents being added, makes 192 *f.* 4 *i.* of one and a half Brick-work, out of which we muſt deduct the Funnel, from the Tarſels to the Top of the Story, which is 1 *f.* × 6 *f.* = 6 *f.* of one and a half Brick-work, becauſe the Funnel, is 14 *i.* by 12 *i.* Well then deduct 6 *f.*

from 192 *f.* 4 *i.* and there remains 186 *f.* 4 *i.* of one and a half Brick-work, for the true Content of theſe two Chimneys in theſe Story; to which muſt be added the Pargetting of the Funnel, which Content found the true Way, is 46 *f.* 4 *i.* of one and a half Brick-work more than the uſual Way, beſides the Allowance to be added for the Pargetting.

An Advertisement.

Becauſe the Pargetting of the Funnels is a Work differing both in Quality and Price from the Brick-work, it will be convenient to find out a Method, that it may be reduced to its Equivalency in Brick-work, which may be done thus; one Yard of Pargetting comes to 4 *d.* and one Foot of one and a half Brick-work, at 5 *l. per* Rod, comes to 4 *d.* 1 *q.* $\frac{64}{100}$, therefore we may allow one Yard of Pargetting againſt one Foot of Brickwork, without any great difference, it being but ſomewhat more than a Farthing, and the Pargetting of a Funnel 1 *f.* in Height is about half a Yard; therefore, when we make Deductions for the Funnels, if we allow but one half of the Deduction, the other half will recompence for the Pargetting, and ſo we need not trouble ourſelves to meaſure the Pargetting any more hereafter in what follows, but uſe this Method of taking but one half of the Deduction.

EXAMPLE IX. Fig. XIV.

Let's try the ſame Stack of Chimneys in the ſecond Story, which we ſuppoſe to be 12 *f.* high: And firſt by the uſual way, we will imagine the Girt to be the ſame as in the Ground Story, *viz.* 21 *f.* × 12 *f.* = 252 *f.* of one Brick-work, which reduced is =

168 *f.* of one Brick and a half Work for the Content of the two Chimneys, according to the uſual Way of Meaſuring? We will try it our Way, the two Jaums added, make 10 *f.* × 12 *f.* = 120 *f.* of one Brick-work = 80 *f.* of one Brick and a half Work: The Back is 4 *f.* × 12 *f.* = 48 *f.* of two Brick and a half Work = 80 *f.* of one Brick and a half Work: The two outſide Breaſts, and inſide Breaſts, and Wings, and Weiths being added together, make 14 *f.* 9 *i.* × 8 *f.* (for we ſuppoſe the Mantle-tree to lie 4 *f.* high from the Floor) produces 118 *f.* of half Brick-work, which reduced is 39 *f.* of one and a half Brick-work, which three Contents being added together, make 199 *f.* of one and a half Brick-work, out of which we muſt deduct the Funnel in each Story; the Funnel of the Ground Story, is 1 *f.* × 1 *f.* 2 *i.* × 12 *f.* = 12 *f.* of one Brick and a half Work, and abating one half of the Deduction for the Pargetting, there remains 6 *f.* of one Brick and a half Work, to be deducted for the Funnel of the Ground Floor in this Story; alſo we muſt deduct the Funnel of this Story from the Mantle-tree, which is 1 *f.* 2 *i.* × 1 *f.* × 8 *f.* = 8 *f.* of one Brick and a half Work, which allowing one half for the Pargetting of the ſame, there remains 4 *f.* of one and a half Brick-work; theſe two Deductions being added, make 10 *f.* to be taken out of 199 *f.* and then there remains 189 *f.* of one Brick and a half Work, the true Content of theſe two Chimneys in this Story, which is 21 *f.* of one Brick and a half Work more than the Contents, by the uſual Way of Meaſuring.

EXAMPLE X. Fig. 5.

The Invention of placing the Funnels without or beyond the Jaums of the Chimneys, as in this fifth Figure,

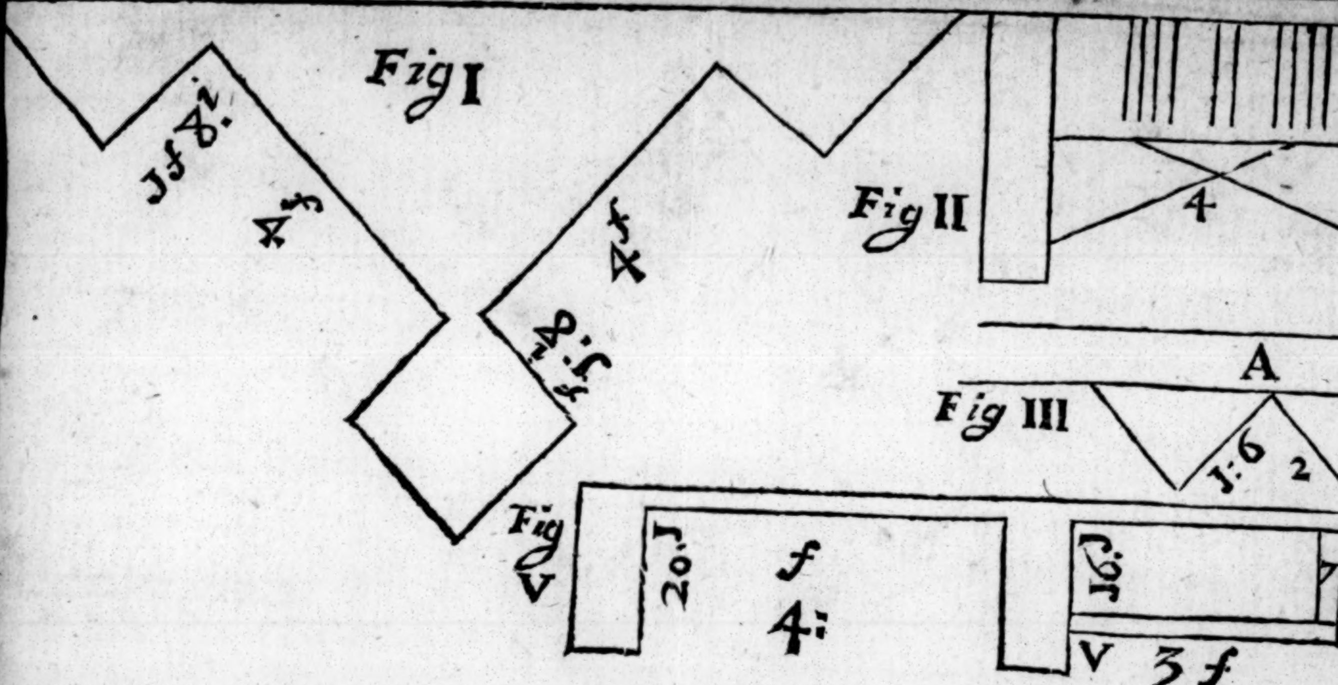


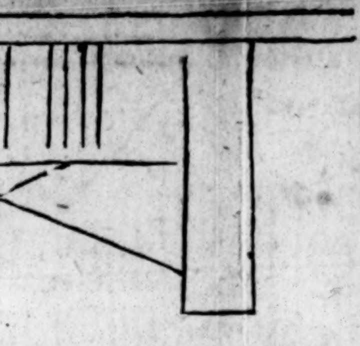
Fig I

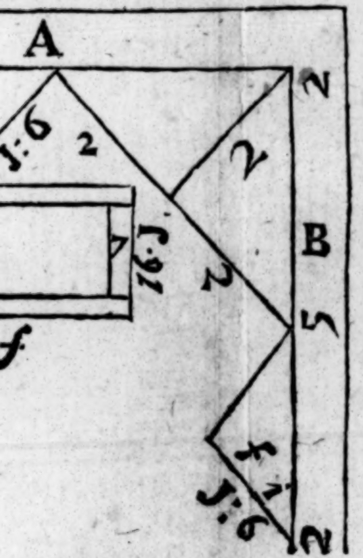
Fig II

Fig III

Fig V

Place this Pa





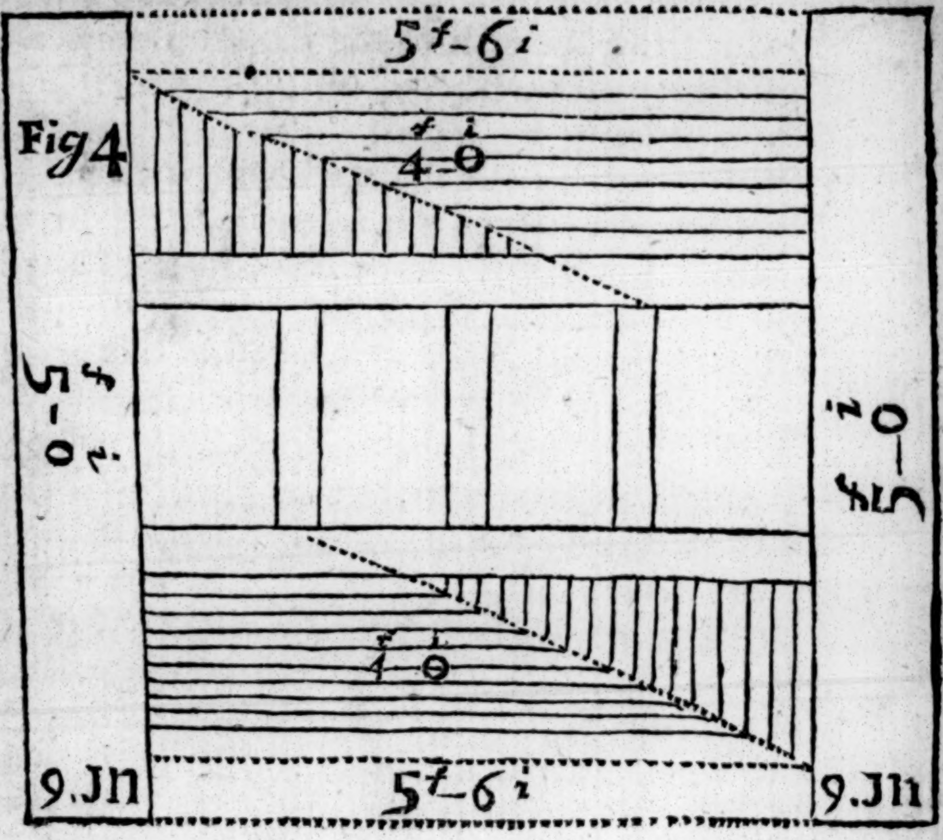


Fig 4

this Page 394.

Figure, hath been but of late Years; and indeed where the Breadth of the Room is but narrow, it is a good Contrivance, by Reaſon the Chimneys do not come ſo far into the Room as they would do, if the Funnels were placed behind them, there being the Breadth of the Funnels, or 16 *i.* ſaved in the Breadth, which is compenſated in the Length: Now to meaſure this Chimney, ſuppoſing the Wall it ſtands againſt, to be either an old Wall, or elſe to have been meaſured before by itſelf; the uſual Way is to girt the Chimney and Funnel about for the Length, and to multiply that Length by the Height of the Story, which we will ſuppoſe to be 10 *f.* the Girt then is 12 *f.* 2 *i.* × 10 *f.* = 121 *f.* 8 *i.* of one Brick-work = 81 *f.* 1 *i.* of one Brick and a half Work. Let's try it our Way, each Jaum is 2 *f.* from the Wall, that is 19 *i.* the Depth of the Chimney, and 5 *i.* for the Breadth of the Back, which two Jaums being added, make 4 *f.* × 10 *f.* = 40 *f.* of one Brickwork = 26 *f.* 8 *i.* of one Brick and a half Work: The Back is 7 *f.* × 10 *f.* = 70 *f.* of half Brick-work = 23 *f.* 4 *i.* of one Brick and a half Work; the Front of the Funnels, is 3 *f.* × 10 *f.* = 30 *f.* of Brick-work 16 *i.* thick, which muſt be reduced to Brick-work of 14 *i.* the uſual Thickneſs, by making it as 14 to 16, ſo 30 *f.* to 34 *f.* 3 *i.* of one Brick and a half Work; the outſide and inſide Breaſts, and Wing, and Wieth, are 7 *f.* 10 *i.* × 6 *f.* (that is from the Tarſels to the Top of the Story) = 47 *f.* of half Brick-work = 15 *f.* 8 *i.* of one Brick and a half Work, all theſe being added together, make 99 *f.* 11 *i.* of one Brick and a half Work for the true Content, it being 18 *f.* of one and a half Brick-work, more than the Content found according to the uſual Way of Meaſuring.

A TABLE of Brickwork *from* 20 s. *to* 50 s. per *Rod.*

	d.	q.	C.
20 *per* Rod, 1 Foot is	0	3	24
21	0	3	68
22	0	3	86
23	1	0	04
24	1	0	23
25	1	0	40
26	1	0	58
27	1	0	76
28	1	0	93
29	1	1	11
30	1	1	28
31	1	1	46
32	1	1	64
33	1	1	81
34	1	1	99
35	1	2	17
36	1	2	34
37	1	2	52
38	1	2	69
39	1	2	87
40	1	3	05
41	1	3	22
42	1	3	40
43	1	3	58
44	1	3	75
45	1	3	93
46	2	0	11
47	2	0	28
48	2	0	46
49	2	0	63
50	2	0	81

From 4 *l.* 10 *s.* to 6 *l.* 10 *s.* *per* Rod.

l.	s.		d.	q.	C.
4	10	*per* Rod, 1 Foot is——	3	3	87
4	11	————————	4	0	05
4	12	————————	4	0	23
4	13	————————	4	0	40
4	14	————————	4	0	58
4	15	————————	4	0	76
4	16	————————	4	0	93
4	17	————————	4	1	11
4	18	————————	4	1	28
4	19	————————	4	1	46
5	00	————————	4	1	64
5	1	————————	4	1	81
5	2	————————	4	1	99
5	3	————————	4	2	17
5	4	————————	4	2	34
5	5	————————	4	2	52
5	6	————————	4	2	69
5	7	————————	4	2	87
5	8	————————	4	3	05
5	9	————————	4	3	22
5	10	————————	4	3	39
5	11	————————	4	3	57
5	12	————————	4	3	74
5	13	————————	4	3	92
5	14	————————	5	0	10
5	15	————————	5	0	28
5	16	————————	5	0	46
5	17	————————	5	0	63
5	18	————————	5	0	81
5	19	————————	5	0	99
6	00	————————	5	1	17
6	1	————————	5	1	34
6	2	————————	5	1	52
6	3	————————	5	1	69
6	4	————————	5	1	87
6	5	————————	5	2	05
6	6	————————	5	2	22
6	7	————————	5	2	40
6	8	————————	5	2	58
6	9	————————	5	2	75
6	10	————————	5	2	93

A TABLE of Tyling, *from* 2 s. 6 d. *to* 40 s. per *Square.*

s.	d.		d.	q.	C.
2	6	*per* Squ. 1 Foot is——	0	1	20
2	7	——————	0	1	24
2	8	——————	0	1	28
2	9	——————	0	1	32
2	10	——————	0	1	36
2	11	——————	0	1	40
3	0	——————	0	1	44
3	1	——————	0	1	48
3	2	——————	0	1	52
3	3	——————	0	1	56
3	4	——————	0	1	60
3	5	——————	0	1	64
3	6	——————	0	1	68
3	8	——————	0	1	76
3	10	——————	0	1	84
4	0	——————	0	1	92
4	6	——————	0	2	16
5	0	——————	0	2	40
5	6	——————	0	2	64
6	0	——————	0	2	88
7	0	——————	0	3	36
8	0	——————	0	3	84
9	0	——————	1	0	32
10	0	——————	1	0	80
11	0	——————	1	1	28
12	0	——————	1	1	76
13	0	——————	1	2	24
14	0	——————	1	2	72
15	0	——————	1	3	20
16	0	——————	1	3	68
17	0	——————	2	0	16
18	0	——————	2	0	64
19	0	——————	2	1	12

The Table of Tyling continued.

s.	d.		d.	q.	C.
20	0	*per* Sq. 1 Foot is	2	1	60
21	0	——— ———	2	2	08
22	0	——— ———	2	2	56
23	0	——— ———	2	3	04
24	0	——— ———	2	3	52
25	0	——— ———	3	0	0
26	0	——— ———	3	0	48
27	0	——— ———	3	0	96
28	0	——— ———	3	1	44
29	0	——— ———	3	1	92
30	0	——— ———	3	2	40
31	0	——— ———	3	2	88
32	0	——— ———	3	3	36
33	0	——— ———	3	3	84
34	0	——— ———	4	0	32
35	0	——— ———	4	0	80
36	0	——— ———	4	1	28
37	0	——— ———	4	1	76
38	0	——— ———	4	2	24
39	0	——— ———	4	2	72
40	0	——— ———	4	3	20

A TABLE *of Perpendiculars for Gable-ends.*

Baſe.		Perpendicular.		
F.	I.	F.	P.	P.
10	0	5	7	6
11	0	6	0	7
12	0	6	8	7
13	0	7	4	9
14	0	7	10	2
15	0	8	4	9
16	0	8	11	6
17	0	9	6	4
18	0	10	0	1
19	0	10	7	8
20	0	11	2	5
21	0	11	9	3
22	0	12	3	9
23	0	12	10	7
24	0	13	5	4
25	0	14	0	2
26	0	14	6	9
27	0	15	1	6
28	0	15	8	3
29	0	16	3	0
30	0	16	9	8
31	0	17	4	5
32	0	17	11	3
33	0	18	6	0
34	0	19	0	7
35	0	19	7	4
36	0	20	2	0
37	0	20	8	9
38	0	21	3	6
39	0	21	10	2
40	0	22	4	3
10	0	05	7	6
9	0	05	0	5
8	0	04	5	8
7	0	03	11	[illegible]
6	0	03	4	1
5	0	02	9	6

OF
MEASURING
Superficies and Solids.

Concerning a Line of SINES *and* CYCLOIDS.

DEFINITION I.

A Line of Sines, is of right Sines, bowing the Termination of the ordinate Applacates to the Arch of the Quadrant. Or it is, A crooked Line deſcribed from the extream Point of the Semidiameter, aſcending by an equal Motion through the bowed Arch of the Quadrant, approaching to the oppoſite Point, according to the Ratio of the verſed Sines of the Arch paſſed over, each Definition ſhall be explained in the firſt Propoſition.

DEFINITION II.

ycloid is the bounding or terminating of the compounded, from the Sines, and from the ſuperior Arches, by the ordinate Applicates to the Diameter of the ſame Circle: Or it is a crooked Line deſcribed from a moveable Point, by a mixt Motion from equal Motions of the Orb, and of the Center: Both ſhall be explained in Prop. 23.

DEFINITION III. Fig. 1.

A Figure is ſaid to proceed by its Elements, in which Lines are taken or accepted parallel to the Baſe, if the Baſe is a Line, or parallel Superficies, if the Baſe is a Superficie.

We will explain it: Let there be a Rectangle MD, take how many ſoever Parallels to the Baſe MN, as PO, *&c.* the Rectangle MD is ſaid to proceed or increaſe by MN, PO, and the other Elements of the ſame Kind: In like manner the Cylinder MD proceeds or increaſes by the Circular Planes MN, PO, *&c.* Likewiſe the Triang. MAN by the Lines MN, PV: The Cone MNA by the Circle MN, PV, *&c.*

DEFINITION IV.

A Figure is Iſoparallel, which proceeds by Elements equal to the Baſe, as a Rectangle, a Cylinder, a Priſm, *&c.*

DEFINITION V.

Homogene Figures are thoſe which proceed by proportional Elements (for Example) the Rectangle MD, and the Cylinder MD, are Homogene Figures, becauſe as the Line MN to PO, ſo the Plane MN to the Plane PO.

DEFI-

DEFINITION VI.

The Center of Gravity is that from which Quantity being hung down by a Perpendicular, 'tis Equiponderate, or in Equilibrium.

DEFINITION VII.

A Ballance is a Line, to the Extreams of which Weights being annexed, and being hung perpendicular from ſome Point of the ſame Line, they make Equilibrium, or weigh equal.

DEFINITION. VIII.

Moments are the Endeavour of Weights hanging, compared to one another.

POSITION I. Fig. I.

Homogene Figures, of the ſame Kind of Altitude and Baſe, are equal; being of the ſame Altitude, they are as their Baſes; being of the ſame Baſe, they are as their Altitudes; being of divers Baſes and Altitudes, they are in Compoſition, or they are compounded of their Baſes and Altitudes: Let there be two Homogene Figures of the ſame Kind, BAC, BCD, of the ſame Baſe and Altitude; they are equal: For as AC is equal to CD, ſo EI is equal to IK, and ſo of the reſt, if there were never ſo many Lines drawn parallel to the Baſes AC, CD; therefore they are equal. Let them be of the ſame Altitude, and not of the ſame Baſe, as ABC, ABD, then they are as their Baſes AD, AC; for as AD to AC, ſo EK to EI, and ſo of the reſt. Therefore ABD is to ABC as AD to AC, that is, as 2 to 1. Let them be of the ſame Baſe, but not of the ſame Altitude, as ABC, AEC, then they are as their Altitudes AB, AE; for as AB to AE, ſo GI to GI, and ſo of the reſt; therefore ABC is to AEC as AB to AE, that is, as 3 to 2. Laſtly, let them be of divers Baſes and Altitudes, as ABD, AEC, then they are in Compoſition, or they are compounded of the Baſes, and of the Altitudes, that is, as the Rectangles under BA, AD, and under EA, AC; *To wit*, AEC is to ABC, as the Rectangle under EAC to the Rectang. under BAC, *viz.* 2, 5 : 6, 25 : : 5 : 12, 5. But BAC is to BAD as the Rectangle under BAC to the Rectangle under BAD. Therefore EAC is to BAD, as the Rectangle under EAC to the Rectangle under BAD, that is in a compounded Ratio, from the Ratio's of AE to AB, and of AC to AD: If the Baſe is a Plane, the Figures will be as Iſoparallels under the Baſes and Altitudes: Alſo the ſame manner of Demonſtration may be applied to other Homogene Figures.

POSITION II. Fig. 2.

A Quadrant is to the Triangle, in Compoſition, or compounded from the Ratio of the Semidiameter to the Altitude, and from the Ratio of the Arch of the Quadrant to the Baſe of the Triangle; Let the Quadrant be ADC, and any Triangle ACH, I ſay, 'tis Homogene to the Quadrant; for as AC to CH, ſo AB to BF: Therefore as AC to AB, ſo CH to BF; but as AC to AB, ſo the Arch CD to the Arch BE; therefore the Figures proceed by proportional Elements, by Defin. 3. therefore they are Homogene by Definition 5. therefore they are compounded of the Baſes and Altitudes, by Poſition 1. and the Baſe of the Quadrant is the Arch of the ſame, but the Altitude is the Radius or Semidiameter of the ſame.

COROLLARY I. Fig. 2.

Hence aſſuming the Baſe of the Triangle CG, equal to the Arch CD, and the Altitude CA; the Triangle ACG, will be equal to the Quadrant ACD: Aſſuming the Baſe leſſer as CK, or greater as CH: ACK will be to ACD, as CK to CG; and ACH to ACD, as CH to CG; laſtly, BCK to ACH, will be as the Rectangle under BCK, to the Rectangle under ACG.

COROLLARY II.

Hence the Rectangle under the Radius AC, and the Half of CG, is equal to the Quadrant ACD; therefore the Rectangle under AC, and the Double of CG, is equal to the Circle; but under four times CG is double of the Circle: Hence laſtly, any Sector, ſuppoſe ACM, is to a Triangle, ſuppoſe ACK, as the Arch CM to CK.

POSITION III. Fig. 3.

A Cylinder is ſeſquialter, or once and a half of an Hemiſphere of the ſame Altitude, and Baſe: Let a

Quadrant be ALB, a Rectangle AL, a Triangle BML, all turned about BL; there will be generated an Hemiſphere from the Quadrant ABL; a Cylinder from the Rectangle AL; a Cone from the Triangle BML; theſe being granted, that begat from the Triline AML, is Homogene to the Cone, begat from BLM; For ſince Circles are as the Squares of their Radii, let any one be EG, the Square of GE or BD is equal to the Squares of GD and GB or GF; therefore that begat from GE equals thoſe begat from GD and GF or DE, therefore that begat from GF, is equal to that begat from DE, therefore that begat from LM is to the begat from FG, as to that begat from DE: Therefore the begat from the Triline AML, and from the Triangle BML are Homogenes by Definition 5. and they are of the ſame Baſe, *viz.* of the begat from LM, and of the Altitude BL, therefore they are equal by Poſition 1. Then ſince that begat from the Triangle BML being a Cone, is one third of the Cylinder begat from the Rectangle AL, that begat from the Triline AML will be likewiſe one third of the ſame Cylinder; therefore that begat from ALB the Quadrant, *viz.* the Hemiſphere, will be equal to two Thirds of the ſame Cylinder, therefore the Cylinder is Seſquialter, or one and a half of the Hemiſphere.

Suppoſe the Radius AB = to 5 *f.* the Altitude AM equal, or = to 5 *f.* the Diameter = 10 *f.* Then for the Solidity of the Hemiſphere, 10 × 10 = 100 × 10 = 1000, the Cube of the Diameter 1000 × 11 = 11000, which divided by 21, gives in the Quotient 524 *fere* for the Solidity of the whole Sphere, the Half whereof 262 *f.* is the Solidity of the Hemiſphere.

For the Cylinder whoſe Diameter is 10 *f.* and its Altitude 5 *f.*

The Area of the Baſe = 78 *f.* 54 × 5 *f.* the Altitude = 392 *f.* 70 for the Solidity of it.

The Cone, the Area of its Baſe 78 *f.* 54 × by $\frac{1}{3}$ of the Altitude 5 *f.* which is 1, 7 *fere* makes for the Solidity of the Cone 130 *f.* 9.

POSITION IV. Fig. 3.

The Superficie of the Hemiſphere is equal to the Superficie of the Cylinder, of the ſame Baſe and Height, the Baſes being taken away, *viz.* the Hemiſphere is Homogene to the Cone: For as that begat from LM to that begat from GF, ſo that begat from the Arch LA, to that begat from the Arch GP, to wit, as the Square of LM to the Square of GF. But the common Altitude is BL, therefore the Figures are as their Baſes; but the Hemiſphere is double of the Cone by Poſition 3. Therefore the Baſe of the Cone begat from LM, is half of the Superficie of the Hemiſphere, begat from the Arch LA: And the Superficie of the Cylinder begat from AM, is double of the Circle begat from LM, *viz.* equal to the Rectangle under AM, and the Quadruple of the Arch of the Quadrant by Coroll 2. of Poſition 2. therefore the Superficie of the Cylinder is equal to the Superficie of the Hemiſphere: Hence the Superficie of the Sphere is quadruple of the Area of the greater Circle.

Example, Diameter 10 *f.* the Periphery is 31 *f.* 417; the Area of the Circle, 78 *f.* 543.

The Area 78, 543 × 2 = 157 *f.* 086 the Superficie of the Hemiſphere.

The Periphery 31, 417 × 5 = 157 *f.* 085, the Superficie of the Cylinder.

Another Way. The Hemiſphere is Homogene to the Cone, therefore the Superficie of it multiplied by one Third of the Height of BL, produces the Solid: But the Cylinder, foraſmuch as it proceeds by Cylindric Superficies, 'tis Homogene to the Triangle; therefore the Superficie begat from MA, drawn into half of ML or BL, produces the Solid; and the Ratio of the Products, or of the Solids is $\frac{3}{2}$, the other Ratio is $\frac{3}{2}$: Therefore the other which is of Superficies is $\frac{1}{1}$, therefore the Superficies are equal.

POSITION V. Fig. 3.

A Sphere being cut by parallel Planes, the Segments of the ſpherical Superficie are as the Segments of the Diameter, cut by the Planes at right Angles. Example, Let the Plane be EK; I ſay the Superficie begat from the Arch AD is to that begat from the Arch DL, as BG to GL, to wit that begat from ABM is equal to the Hemiſphere, by Poſition 3. And that begat from ABE proceeding by Cylindric Superficies, of which the firſt is begat from AE, is Homogene to that begat from ABD proceeding by Sphericals, of which the firſt is that begat from AD: And ſince that begat from GF is equal to that begat from DE, and ſince that begat from BFG is to that begat from BED, as the begat of the Baſe from FG to that begat from DE: the begat from BFG, will be equal to that begat from BDE: But that begat from BFG is equal to that begat from the Triline AED; and by taking away the common begat by the Triline VED, the Remainers will be equal, *viz.* the begat from BVD, and AEV, therefore the begat from BAD is equal to that begat from BAE, but theſe are Homogenes of the ſame Altitude AB, and therefore of the ſame Baſe, by Poſition 1. And the Baſes of the begotten from AE, and from the Arch AD: But that begat from AE, is to that begat from EM, as AE to EM, or as BG to GL; therefore that begat from the Arch AD, to that begat from the Arch DL, as BG to GL; the ſame being aſſumed, it may be demonſtrated in any other Point: Therefore the Segments of the Superficie of the Hemiſphere, are as the Segments of the Semidiameter for the ſame Planes; therefore the Segment of the Superficie of the Sphere, are as the Segments of the Diameter.

POSITION VI. Fig. 4.

If as AD the Radius of the Quadrant ADK, or the whole Sine, to DI the right Sine of the Angle BAL, ſo DI to DC; AD will be to AI, as AI to AC, of which alſo AD, the Difference is the ſame DC, which is the verſed Sine of the Angle DBI, the double of DAI in the Quadrant DBM, under the Radius BD ſupduple, or half of the former AD; to wit, the ſubtenſe DH, and all the other will be cut in the Middle by the Periphery DMA: Theſe Things appear from the Elements.

POSITION VII. Fig. 5.

If there be any Fig. ſuppoſe the Semicircle AHM, which let be turned about AC; that begat from AMH is to that begat from the Rectangle AL, as the Iſoparallel under the Baſe AMH, and from the Altitude AM, to the Parallelipiped under the Baſe AL and the Altitude AM, to wit, that begat from AM is to that begat from DI, as the Square of AM to the Square of BI, leſs by the Square of BD, that is to the Rectang. under the Baſe DI, and the Altitude AM. The ſame may be demonſtrated by aſſuming any other. Therefore that begat from AMH is to that begat from AL, as the Iſoparallel under the Baſe AMH, and the Altitude AM, to the Parallelipiped of the ſame Altitude under the Baſe AL. If we aſſume any other Fig. (for Example) the Triang. AMH, or any other Triline, the ſame may be demonſtrated generally.

POSITION VIII. Fig. 6.

Homogene Figures of the ſame Altitude have the Center of Gravity equidiſtant from the Baſe. Let CAB, EAB be homogene Figures, and let DA be the Diſtance of the Centre of Gravity of the Fig. CAB from the Baſe AC; it will be the ſame of the Center of the Fig. EAB from the Baſe AE. For if to the Perpendicular DF you hang down CAB, it will equiponderate. Therefore the Moments of the Trapezium ACFD, and of the Triang. DFB, are equal; but they are compounded of the Quantities and of the Diſtances by exchanging. And ſince the Trapezium ADGE is to DGB the Triang. as ADFC is to DFB, the Center of Gravity of DFB will be in OL; and it will be, as the Trapezium ADFC to DFB, ſo OD to DI; the Center of the Trapezium will be in IH. Therefore alſo the Center of GDB in OM, and of the Trapezium ADGE in IK. Therefore of the whole Fig. ABE the Center of Gravity will be in DG. The ſame may be demonſtrated in any other Homogenes, either Solids or Planes. I ſhew theſe in haſte, and briefly, which, in *Cavalerius* his 5th *Exerciſe*, at Propoſition the 9th, and others that have writ concerning *Staticks in Geometrical Rigour*, may be found demonſtrated more at large. Therefore ſince theſe things are ſuppoſed, it will ſuffice to ſhew them briefly.

POSITION IX. Fig. 7.

If two Figures (for Example) EA, BAC, do weigh on the common Axis BA, the Moments will be as the Solids begat from the ſame, being turned about BA; becauſe the Moments are compounded from the Ratio of the Quantities, and from the Ratio of the Diſtances of the Center of each Figure from the common Axis BA. Therefore the Moment of KG is to the Moment of GI, in the compounded Ratio of the whole to the whole, and of the $\frac{1}{2}$ to the $\frac{1}{2}$; that is, in duplicate of GK to GI; that is, as the Square of GK to the Square of GI; that is, as that begat from GK, to that begat from GI. The ſame may be ſhewn by aſſuming any other. Therefore the Moment of the whole EA, is to the Moment of BAC, as that begat from IA to that begat from BAC. Theſe things I ſhew briefly; they may be ſeen demonſtrated more at large in the aforeſaid *Cavalerius*, and in *Torricellius*, concerning the meaſuring of the *Parabola*, *Lem.* 31.

COROLLARY.

Hence from a given Ratio of the Figures, and of the Diſtances from each Center (for Example) of GF, GH, the Ratio of the Solids may be known, or of the Generators or Begetters from the ſame Figures; and back again from the Ratio given of the Solids and of the Diſtances, the Ratio may be had of the Figures; and from the Ratio given of the Solids and of the Figures, the Ratio of the Diſtances is had.

PROP. I. Fig. 8.

An homogene Figure of a Figure of Sines, the Baſe of which is quadruple of the Axis, equals the Superficie of the Hemiſphere under the Radius, equal to the Baſe of the Fig. of Sines. That this Propoſition may be underſtood, ſomewhat of Conſtruction muſt be uſed. Let BAE be at right Angles, and let AE be quadruple of AB; let any Quadrant be APL; divide the Arch of it AL in the middle in S, the Sine SZ. Divide AB in the middle in D, and let it be as the whole Sine AP to SZ, ſo AE to the Ordinate applied DH. In the ſame manner may be found the other Ordinates applied from the Arch AL and the right Line AB proportionally divided. Laſtly, through the Extreams of the Applieds ſuppoſe the Curve Line BHE to be drawn: Let AQ be equal to AB, and let it be as AE to

D B, ſo AE to DK; and do the ſame in the other Applieds, the Fig. ABKQ will be homogene of the Fig. ABHE. Laſtly, let it be as AC to AB, as Radius, or the Semidiameter to the Arch of the Quadrant; let AP be equal to AC, and let DI be equal to SZ; and in like manner apply the other Sines; BTP will be the Line of Sines; and call ABP the Fig. of Sines, AB the Axis, AP the Baſis, APL the generating Quadrant, CBT the right Segment, YTP the verſed Segment, CT, DI Ordinates applied; T Υ parallel to the Axis, ACTP the right Trapezium of the Fig. ABT Υ the verſed Trapezium; that begat from the Fig. about BA the right Solid; that about AP the verſed Solid, and it will appear that AE, the Quadruple of AB, is equal to the Periphery of the Circle under the Radius AC or AP: Alſo it appears, that the Fig. ABE is homogene of the Fig. of Sines: Laſtly, turn about PA in the Plane of the Quadrant, the Point A will deſcribe the Arch ASL, equal to the right Line AB, which if it be turn'd about LP, that A will deſcribe the Periphery under the Radius PA; ſo S will deſcribe his under the Radius SZ, and each other Point his under the Radius which is the Sine terminated at the generating or begetting Point: and at length the whole Arch ASL, the Superficie of the Hemiſphere. Theſe things being done, the Propoſition may be eaſily demonſtrated.

For the Fig. ABHE is homogene of the Fig. of Sines ABTP, by *Defin.* 5. the Baſe AE is quadruple of the Axis AB; and it is as AP to SZ, ſo AE to DH; AE is equal to the Periphery under the Radius AP of the Begat, *viz.* from A. Therefore DH is equal to the Periphery under the

Radius SZ of that Begat, *viz.* from S; therefore the Fig. ABE, and the Superficie of the Hemiſphere begat from the Arch ASL, are homogene Figures by *Defin.* 5. They are of the ſame Baſe, for AE is equal of the Periphery of the Begat from A; alſo of the ſame Altitude, for the Arch AL is equal to the right Line AB: Therefore the Figures are equal by *Poſition* 1.

PROP. II.

The Fig. ABHE is to ABKQ, as AE to AC; likewiſe to the Fig. of Sines as AE to AP, that is, as the Periphery to the Radius; becauſe, ſince they are homogene by Conſtruction, and of the ſame Altitude AB, they are as the Baſes by *Poſition* the 1ſt. Therefore as AE to AQ and AB. The ſame may be demonſtrated concerning all other Homogenes.

PROP. III.

Any Fig. homogene of the Fig. of Sines ABTP. and of the ſame Altitude AB, is equal to the Rectang. under its Baſe and AC or AP: For ſince ABHE is equal to two Circles under the Radius AC or AP, by *Poſition* 4. it will equal the Rectang. AF by *Corol.* 2. of *Poſition* 2. Therefore ſince ABHE is to ABTP as AE to AP, by *Prop.* 2. it will be as the Rectang. AF to the Rectang. AL. Therefore ABTP is equal to the Rectang. under the Baſe AP, and AC. In like manner it may be ſhewn, that ABKQ, is equal to the Rectangle AM; and ſo of the reſt.

COROL. I.

Hence the Fig. of Sines is equal to the Quadrate of its Baſe, or to the Radius of the generating Quadrant, ſo ABTP is equal to the Quadrate AL, for AL is the Quadrate, becauſe AP is equal to AC.

COROL. II.

The Triline BMP is equal to the Difference of the Square of the Radius, to wit, of AL, and of the Semicircle under the ſame Radius; for AM is equal to the Semicircle by *Corol.* 2. of *Poſition* 2. and ABTP is equal to the Square of the Radius.

COROL. III.

Hence the aforeſaid Triline is equal to the Rectangle EM under the Radius, and the Difference of the ſame Radius, and the Arch of the Quadrant.

COROL. IV.

The ſame Triline is equal to twice the Segment of the Circle under the Radius AP, contained by the Arch of the Quadrant and the Subtenſe.

COROL. V.

The Segment contained by the right Line BP, and the curve Line BTP, that is the Line of Sines, is equal to the Triline ACL.

COROL. VI.

As AO the Square of the Arch of the Quadrant to AM, or the Semicircle; ſo AM to AL the Square of the Radius: Hence the Semicircle is a mean Proportional between the Square of the Radius, and the Arch of the Quadrant.

COROL. VII.

The Triline CBP equals the Triline ACL: Likewiſe BCT equals the Triline PLT: Alſo BCY equals the Section under the Arch YP, and the right Line YP.

SCHOL.

We muſt imagine in our Mind the Arch ASL unbent from Curvity, and deflected into the right Line AB; likewiſe each Periphery ordinately applyed at right Angles to the Arch ASL (for ſo the Radius falls in with the Arch) being deflected alſo into right Lines equal to AB, ordinately applyed at right Angles: So the whole Superficie of the Hemiſphere from the Arch ASL begat in the Plane Fig. goes ABHE away, and the Quantity remains the ſame, but the Round only and the Curve are changed into right Lines and Planes.

PROP. IV. Fig. 9.

If the Fig. of Sines be turned about the Axis, that begat or generated is ſubduple of the Cylinder of the ſame Baſe and Altitude: Let the Fig. of Sines be ABC, the Quadrant ADC, the Arches DG, IC are equal: And let the Sine EG be tranſlated into OR, and KI into LV; OB will be equal to AL, alſo to the Arch DG or IE: And ſince the Square of AC or AI is equal to the Squares of KI, and IC, or EG, that is the Squares LV and OR; the Difference of the Squares LV, LM, will be equal to the Square of OR, and the Circles are as the Squares of the Radius's; therefore that begat from LV, with that begat from OR, is equal to that begat from VM: In like manner that begat from RS, is equal to that begat from LV or Z; therefore that begat from ABC, and from the Triline BSC are homogene; therefore they are equal ſince the Baſes are equal, and are in the ſame Altitude; therefore either of them is ſubduple to that begat from AS, to wit, of the Cylinder of the ſame Baſe and Altitude.

COROL. I.

Hence the Semiparabola under the Axis AB, and the Baſe AC, is indeed greater than the Fig. ABC, yet being revolved about the Axis, begets a Solid equal to that begat from the Fig. ABC: But if AB be divided in the middle in N, and NP be drawn, the aforeſaid Semiparabola will cut the Line of Sines in PI; and the Applicates or Applyeds above NP in the Parabola are greater, but beneath leſſer than in the Line of Sines.

COROL. II.

That begat from any Fig. homogene of the Fig. of Sines, is ſubduple of the Cylinder of the ſame Altitude and Baſe, for they are Homogenes begat of homogene Figures.

COROL. III. Fig. 8.

That begat from the Quadrant ABQ, to that begat from the Fig. ABKQ, is as 4 to 3, becauſe that begat from the Quadrant, to wit, the Hemiſphere, is to the Cylinder of the ſame Baſe and Altitude, as 4 to 6.

COROL. IV. Fig. 8.

That begat from the generating Quadrant ALP or ACP, is to that begat from the Fig. ABP, is as half of the Radius CA to two Thirds of AB, equal to the Arch of the Quadrant ASL; that is according to *Archimedes*'s Cyclometry, as 28 to 3: But to that begat from the Triline CBP, as one Third of AC, to the Difference of two Thirds of AC and two Thirds of AB, that is as 28 to 5.

COROL. V.

If there be a mean Proportional between AB, AP, the Parallelipiped under the Square of it, and of the Altitude AB, will be equal to that begat from the Fig. ABTP; for it is ſubduple of the ſame Cylinder, ſince the Baſe of it is equal to the Semicircle.

COROL. VI.

That Begat from homogene Figures ABTP of the ſame Height or Altitude, are as the Square of the Baſes of the Figures, (for Example) that begat from ABTP is to that begat from ABKQ, as the Square of AP to the Square AQ.

COROL. VII.

That begat from the Lunula contained by the Arch of the Quadrant BVQ, and the Curve Line BKQ, is ſubduple of the begat from the Triangle ABQ: For let that begat from the Rectangle AO be 6, that begat from the Triangle ABQ will be 2; from the Fig. ABKQ, 3; from the Quadrant ABVQ, 4; therefore that begat by the Lunula will be 1: Therefore the Subduple of that begat from the Triangle ABQ, and equal to the begat from the Segment contained by the Curve Line BKQ, and the right Line BQ: Hence as the Superficie begat from the right Line BQ, will divide in the middle that begat from the Quadrant, *viz.* the Hemiſphere; ſo the Superficie begat from the Curve Line BKQ, will divide in the middle the Begat from the Segment of the Quadrant contained by the Arch BVQ, and the right Line BQ.

COROL. VIII. Fig. 10.

If you let the Quadrant be ATMB, and the Fig. homogene of the Fig. of Sines ATNB, the Rectangle AG, AT divided in the middle in

C;

C; CF Parallel to AB, the right Line begat from the four Segments CDEOF are equal: For let that begat from CF be 16, that begat from CD will be 4; from CE, 8; from CO will be 12; therefore from DE, 4; from EO, 4; from OF, 4: And therefore equal.

COROL. IX.

If you draw two Lines RP, LH Parallels to EF, and equidiſtant from the ſame, ſince that begat from RS and IH are equal, for AL, RT, RS are equal, likewiſe thoſe begat from LK, MP will be equal to the begat from RM and KH: And by ſubtracting the Equals begat from RS and IH, the Remainers will be equal, to wit, the begat from SM, KI: The ſame may be demonſtrated from any other being aſſumed: Hence from the Cone begat from ATB being ſubtracted from the Hemiſphere, the Remainer will be divided in the middle by the Plane CF.

COROL. X. Fig. 9.

If AL, OB are aſſumed equal, the Cylinder under the Baſe from the Circle from AC, and from the Altitude AL, is equal to the begat from the right Segment OBR, and from the Trapezium ALVC, to wit, the begat from the Triline CVM is equal to that begat from the Segment OBR.

COROL. XI. Fig. 9.

Hence that begat from LVRO is to the Remnant as NL to LA: Hence may be had a mean

Fruſtum in any given Ratio to the whole; if the ſeſquitertia Ratio be required, let N L be to N A, as 3 to 4, the begat from L V R O will be to the begat from the Fig. A B C, as 3 to 4, that is N L to N A.

COROL. XII.

Laſtly, hence that begat from OBR, and CMV are equal, likewiſe that begat from A L V C, and from BR γ S; alſo the begat from RTY δ, and from L V θ T are equal, *&c.*

PROP. V.

Any right Segment of a Fig. of Sines is to the whole Fig. as the verſed Sine of the Arch of it, to which the Altitude of the Segment is equal to the whole Sine: For let any Segment be N B P whoſe Altitude NB is equal to the Arch DH, and therefore HF equal to N P: I ſay the Segment NBP is to the whole Fig. ABC, as the verſed Sine of DFI to the whole Sine D A: For ſince the Fig. A B C is homogene to the Superficie of the Hemiſphere, this will be divided in the Ratio of the Segments of the Radii, by Poſition 5. (for Example) If you revolve the Arch D C about D A, the Superficie begat by the Arch DH, is to that begat from the Arch D C, as the Segment N B P to the Fig. ABC: But that begat from the Arch D H is to that begat from the Arch DC, as DF to DA by Poſition 5. Therefore the Segment N B P is to A B C, as D F the verſed Sine of the Arch D H, equal of the Altitude of the

Segment N B to D A the whole Sine. The ſame may be demonſtrated in any other Segment.

COROL. I.

Hence may be given the Segment which is to the whole Fig. in any given Ratio, (for Example) let the Ratio given be of D E to D A, let O R be applied equal to E G, the Segment O B R will be to the whole A B C, as D E to D A: Hence if a Segment be required which ſhall be one quarter of the whole, let D F (for Example) be one quarter of D A, let N B be applyed equal to F G; N B P will be to A B C, as 1 to 4: Hence if A T be half of A B, T B θ will be one Third of ABC; and therefore T θ will divide the Fig. of Sines equally.

COROL. II.

Hence the right Segment is to the whole Fig. as the Applyed to the Triline of the Fig. conjoined equally diſtant from the Baſe of the Triline, alſo the Baſe of the Segment from the Baſe of the Fig. O A will be equal from the Baſe of the Triline S M; I ſay the Segment O B R is to the whole ABC, as the Applyed to the Triline V M to the Baſe B S: For let the Sine E G or Z I be equal to O R, and therefore O B equal to the Arch D G or C I, the Segment O B R is to the whole A B C, as D E or Z C, to D A or A C: But L V is equal to A Z or A E; therefore VM is equal to Z C; therefore O B R is to A B C, as V M to B S: The ſame may be demonſtrated concerning any other Segment.

PROP.

†

PROP. VI. Fig. 11.

The Ratio may be defined of the verſed Sine to the whole Fig. of Sines : Let the Fig. of Sines be APT, the Quadrant AMT, the Triline annex'd PVT, let AK be equal to AT, and therefore the Quadrate AN ; let any verſed Segment be BRT, draw LR indefinitely, let LR be equal to IF ; laſtly, through the Point X, in which BR the Axis of the Segment will cut KN the ſide of the Quadrate AN, draw AXS from S, let fall SC parallel to the Axis RB ; ſince the whole Fig. APT is to the right Segment LPR, as AT to AC, it will be to the Remnant, to wit, to the verſed Segment BRT, as AI to CI ; therefore the Ratio ſought is found.

COROL. I.

Hence the whole Fig. is to each Segment ART, LPR, as AT to CT.

COROL. II.

Hence the whole Fig. is to the verſed Segment, as the whole Sine to the Baſe of the Segment leſs by the Compound, whereby the whole Sine exceeds the right Sine of the Arch equal to the Axis of the Segment, and from the Line which is to the Exceſs, whereby the aforeſaid Arch exceeds the Radius, as the Sine of the Complement of the aforeſaid Arch to the Radius.

PROP.

P R O P. VII. Fig. 12.

If a Fig. of Sines be turned about a Parallel to the Axis erected in the Extream of the Baſe, that begat from the Fig. will be to the Cylinder of the ſame height under the Circle of the Baſe, to that begat from the Fig. of the Baſe, as the Compound from the Radius, and the Exceſs whereby the Radius exceeds half the Axis to the whole Axis: For let the Fig. of Sines be A B C inſiſting perpendicularly to the Plane of the Quadrate A E, and let the Solid proceed by the Squares A E, N I, *&c.* under the Baſe AE, and the Altitude AB, and therefore homogene to that begat from the Fig. A B C turned about the Axis A B, it will make the Iſoparallel under the Baſe A B C, and the Altitude A D, to wit, proceeding by the equal Planes A B C, D K E: Let A X be equal to A C, likewiſe A N equal to B N, and X V equal to X N; the Iſoparallel aforeſaid will be to the Parallelipiped A F, to wit, under the Quadrate AE, and the Altitude AB, as A B C to the Rectangle A L; for they are homogene Iſoparallels, therefore if they ſhall be of the ſame Altitude, they are as their Baſes by Poſition 1; therefore as A X to A B, the aforeſaid homogene Solid proceeding by the Squares A E, N I, is to the Parallelipiped A F, as A N to AB, by Prop. 4. therefore it is to the aforeſaid Iſoparallel, as AN to AX; therefore to the Difference of each, as A N to N X: And by taking away from the Parallelipiped A F the Solid proceeding by the Squares B F, I H, to wit, under the Applicates to the Triline KFE; and by taking from the Reſidue, the aforeſaid Homogene proceeding by the Squares AE, NI, under the Applicates of the Fig. the Reſidue will proceed by Supplements parallel to the Gnomons IG, IS : For as from the Square NH IH being taken away, and from the Reſidue NI being taken away, there will remain the Supplement of the Gnomon IG, IS : So it may be done in any other Square aſſumed ; and ſince IG is equal to IS, and that from any other Quadrate aſſumed, the Exceſs whereby the Reſidue of the Parallelipiped by taking away the Solid, proceeding by the Squares BF, IH, will exceed the Iſoparallel under the Baſe ABC, and the Altitude AD, equal to the Exceſs whereby the aforeſaid Iſoparallel exceeds the aforeſaid Homogene proceeding by the Squares AE, NI : Therefore this Homogene it ſelf is to the aforeſaid Iſoparallel as AN to AX, and to this the Difference of both being added, as AN to AV ; laſtly to the Solid, proceeding by the Squares BF, IH, as AN to VB.

But ſince the Solid proceeding by the Squares BF, IH, is homogene to that begat from the Triline KFE, or BLC turned about LC ; this begat will be to the Cylinder of the ſame Baſe and Altitude, as BV to BA ; therefore that begat from BAC, turned about CL, will be to the ſame Cylinder, as AV to AB ; therefore as the Compound from the Radius AX and XV, to wit, the Exceſs whereby the Radius AX exceeds AN the half of the Axis AB, for XV is equal to XN.

COROL.

COROL. I.

Theſe may eaſily be reduced to Calculation according to the Cyclometry of *Archimedes*, for AN will be to AX as 11 to 14; and to AV as 11 to 17; and to VB as 11 to 5: Hence if the Cylinder begat from AL turned about CL be 22, that begat from the Triline BCL will be 5, and from the Fig. AB 17; and the Parallelipiped AF, to the Iſoparallel under ABC and AD as 22 to 14.

COROL. II.

If between NG, NP, there be a mean Proportional, and alſo between the Sides of other Rectangles Parallels of NM: The ſolid Proceeding by the Squares under the aforeſaid mean Proportionals, is equal to the Iſoparallel under ABC and AD: But if from the Applicates ordinately applied to BA, by the aforeſaid mean Proportionals, the Fig. turned about AB, that begat will be to the Cylinder of the ſame Baſe and Altitude as AX to AB, that is, as 14 to 22.

PROP. VIII. Fig. 13.

If the Fig. of Sines is CAE, and the other common Axis EAB, and let the ſolid Iſoparallel under EAB, and the Altitude AI be equal to AE: And laſtly, the Solid be cut by the Plane BIE, the lower Fruſtum RIQB is to the whole Solid, as the Radius CE to the Axis CA; but the upper Fruſtum, as the Difference of the Axis, and of the Radius to the Axis; and to the other Fruſtum as the aforeſaid Difference to the Radius: We will demonſtrate it; let I T be equal to the Radius E C, and let the Fig. of Sines be T O I, and the accompanying Triline O A I, the ſuperior Fruſtum E A B I will proceed by right Segments the Parallels B A E, N D M, &c. And B A E is to N D M, as A O to D F, by Coroll. 2. of Prop. 5. The ſame may be demonſtrated concerning any other Segment; therefore the Fruſtum E A B I is Homogene to the Triline A O I; therefore as the Triline A O I to the Rectangle A T, ſo the Fruſtum E A B I to the Iſoparallel under the ſame Baſe E A B, and the Altitude A I: But the Rectangle A T is to the Triline A O I as the Axis A I to the Difference of the Axis A I or A C, and of the Radius C E: Therefore the aforeſaid Iſoparallel is to the aforeſaid Fruſtum in the ſame Ratio, therefore to the Fruſtum R I Q B as the Axis to the Radius; therefore the Fruſtum R I Q B, to the Fruſtum E A B I, as the Radius to the aforeſaid Difference.

C O R O L. I.

Hence the Fruſtum R I Q B, ſo far as it proceeds by right Trapeziums, parallels to the Plane R I Q, 'tis Homogene of the Fig. of Sines T O I; to wit, as T I to π F, ſo R I Q to Z M N V.

C O R O L. II. Fig. 9.

Hence any right Trapezium is to the whole Fig. as the Applicates of the Fig. equally diſtant from the Baſe of the Fig. and of the Baſe of the right Segment joined to the Trapezium, diſtant from the

Vertex of the Fig. to the Radius; (for Example) let the right Segment be OBR, let LV be applied equally diſtant from AC, as OR from the Vertex B, the Trapezium AORM will be to the whole ABC, as LV to AC; becauſe ſince the whole ABC is to OBR, as LM to VM, it will to the Remnant, *viz.* AORC, as LM to LV.

COROL. III. Fig. 13.

Hence as the Rectangle TA will be divided by the Curve OIFI, ſo the aforeſaid Iſoparallel by the Plane BIE; therefore according to *Archimedes*'s Ratio, the upper Fruſtum is to the lower, as 4 to 7; but to the whole as 4 to 11; but the lower to the whole, as 7 to 11.

PROP. IX.

The Center of Gravity of the whole Fig. of Sines, or of the double, with the common Axis, will divide the Axis ſo, that the Segment towards the Vertex of the Fig. is to the whole Axis, as the Radius to the Arch of the Quadrant: Let the Fig. of Sines be CAE, with its ſociate CAB, the common Axis AC, ſo divided in K, that the Segment KA be made to the whole CA, as the Radius to the Arch of the Quadrant; K will be the Center of Gravity of the Fig. EAB; that it will be in the Axis AC, appears, ſince AC divides in the Middle all the Applicates EB, HG, &c. which is in K, ſhall be demonſtrated: Let fall KX parallel to AI divided in the Middle in S, and draw Sδ parallel to AC; draw Iδ, SC; theſe are parallel, in Iδ is the Center of Gravity of the Fruſtum RIQB, becauſe it will paſs by the Center of Gravity of all the Rectangles of the Parallels ER: Alſo in SC will be the Center of Gravity of EABI becauſe it paſſes through the Center of Gravity of all the rectangled Parallels HA. For Example, let the Center of the Fruſtum EABI be in P, and let the Point Y be in which S ♂ cuts KX, and draw PY θ; ſince the Triangles SPY, ♂ θ Y are Proportional, it will be as PY to θ Y, ſo SY to Y ♂, that is, AK to KC; but PY is to Y θ, as the Fruſtum RIQB to the Fruſtum EABI; for the Diſtances are as the Weights by exchanging: Therefore AK is to KC as the lower Fruſtum to the upper, therefore to AC as to the whole; therefore as the Radius to the Arch of the Quadrant. Now this hath been demonſtrated in general; concerning each Iſoparallel you may conſult *Torricellius* with *Cavellerius* at Exerciſe the 5th, Propoſition the 17th.

COROL. I.

Hence the Center of Gravity of the Iſoparallel will be Y; becauſe KX will be drawn through the Centers of all the parallel Planes BAE, RIQ, therefore it will be in KX: Moreover, S ♂ will paſs through the Center of Gravity of all the parallel Planes of BQ; therefore it will be in S ♂, therefore in Y.

COROL. II.

Hence the Center of Gravity of the Fig. of Sines CAE, is in the right Line KH, as appears: Likewiſe of the Fig. CAB in the right Line KG, to wit, the Center of both equally diſtant from EB, or from the Vertex A, ſince the whole EAB is Homogene of each aſunder.

PROP. X. Fig. 14.

The Center of Gravity of the Fig. of Sines is diſtant from the Axis one quarter of the Axis, and from the Baſe, by the Exceſs it ſelf, whereby the Axis exceeds the Baſe: Let the Line of Sines be AMC, with the Rectang. CY; the Moments of both Figures librate in CM, they are as the Solids begat from the Figures turned about CM, by Poſition the 9th. And that begat from CY is double of that begat from ACM by Prop. 4. but the Ratio of Solids begat from Figures is compounded from the Ratio of the Figures, and from the Ratio of the Diſtances of the Center of both from the common Axis by Poſition 9; therefore the Moment of the Rectangle CY is to the Moment of the Fig. CMA, in the Compoſition from the Ratio CM to CO, or its equal MD, which is the Ratio of the Figures, and from the Ratio of EQ, or ST, the half EX; for this is the Diſtance of the Center Q from the Axis CM, to the Diſtance of the Center of the Fig. CMA from the ſame Axis CM; that is, the Moment of the Rectangle is to the Moment of the Fig. as the Rectang. under CM, CT to the Subduple Rectangle, under one of the Sides equal to CO, and under the other, which may eaſily be found by drawing OR, alſo CR, PVS; for CP is equal to CQ, and EV is one quarter of MC; for ſince EP, CQ are equal, as CO to CE, the half of CM, ſo CT, the half of CO to CS, the half of CE; therefore CS or EV is half of CE; therefore one quarter of CM; therefore CS is the other Side of the Rectang. ſought, therefore 'tis the Diſtance of the Center of Gravity of the Fig. CMA from the Axis MC. Laſtly, ſince CD is the Exceſs whereby the Axis CM exceeds the Baſe CA, by drawing DF parallel to the Baſe, and aſſuming DF equal to CS, the Center of Gravity of the Fig. CMA is in F.

COROL. I.

Hence ſince ME is to DE, as 4 to 1, that is, as the Rectang. CY to the Rectang. under one quarter of CM, and under CA, that is, as the Semicircle to the Semiquadrant, CA will be to DF as the Square of the Radius to the Semiquadrant: Therefore equal weighing being made, and aſſuming the Weight to the Arm CX, with the Perpendicular CM, the Semiquadrant will equiponderate the Fig. CMA, for the Moments are equal, *viz.* compounded from the Ratio of CX to CY, and from the Ratio of CY to CX.

COROL. II.

Hence is had the Center of the Triline MKA, *viz.* by drawing EG equal to EQ, and through G draw FGH: It will be as CD to DM, ſo FG to GH; H will be the Center ſought, and the thing may eaſily be reduced to Calculation; let CM be 11, CA will be 7, likewiſe CO, MD will be 7, therefore CD will be 4, DE will be $1\frac{1}{2}$; EV $2\frac{5}{8}$; NM, or IK, $2\frac{7}{8}$; LK will be $2\frac{3}{10}$.

COROL.

COROL. III. Fig. 13.

Hence the Center of Gravity of each Fruſtum of the Iſoparallel in Prop. 8. may be found; for ſince the upper Fruſtum EABI is Homogene to the Triline OAI, by Prop. 8. the Center of the Fruſtum will be equally diſtant from the Baſe EAB, as the Center of the Triline from the Baſe AO: And theſe Diſtances are had by Coroll. 2. let it be AD, draw DP parallel to AC; I ſay, the Center of the Fruſtum is in P; for ſince 'tis in CS, alſo in DP, it will be in P, in the Point of the common Section. In like manner, the Center θ of the other Fruſtum is had, as well by drawing the right Line P θ through Y, as alſo by the ſame Method, ſince this Fruſtum is Homogene of the Fig. of Sines TIQ, by Coroll. 1. Prop. 8.

COROL. IV.

Hence may be known the Quantity of the Line DP; for ſince AC is to AS, as DP to DS; and ſince AC is double of AS, DP will be double of DS: Therefore by putting the Axis AI to be 11, DS will be 2 $\frac{5}{8}$, and conſequently DP will be 5 $\frac{1}{4}$ becauſe 'tis the Double of 2 $\frac{5}{8}$, or of DS.

PROP. XI. Fig. 14.

If the Fig. of Sines be turned about the Baſe, that begat from it is to the Cylinder of the ſame Baſe and Altitude, as the Rectang. under the Baſe,

and

and the Exceſs whereby the Axis exceeds the Baſe; to the Rectang. under the Axis and half of the Axis; for let the Fig. be CMA, the Rectang. CY; turn both Figures about AX, the Solids begat are in Compoſition from the Ratio of the Figures, which is CO to CM, and from the Ratio of the Diſtances of the Center of both from AX, which is CD to CE by Poſition 9. and Prop. 9. Therefore that begat from CMA is to that begat from CY, as the Rectang. under CO, CD, to the Rectang. under CM, CE. Therefore as the Rectang. under the Baſe, and the Exceſs whereby the Axis exceeds the Baſe, to the Rectang. under the Axis, and half the Axis.

COROL. I.

Hence is had that begat from the Triline MAK; for by taking away that begat from CMA being known, from that begat from CY being known, the Reſidue will be the begat from the Triline MKA alſo known; and that the thing may be reduced to Calculation, let that begat from CY be 121, that begat from CMA will be 56; therefore that begat from the Triline MKA will be 65.

COROL. II.

And that we may find that begat from the Fig. CMA turn'd about MK, let that begat from the Fig. CMA turn'd about CA, be an Homogene Solid under the Square of the Baſe CM, and the Altitude CA be called B: Let the Parallelipiped of the ſame Baſe and Altitude be called C: Let the

Iſoparallel under the Fig. of the Baſe CMA, and the Altitude CM be called D; then C will be to D, as MC to MD: Add to D the Exceſs whereby D exceeds B; C will be to the Aggregate from D, and the aforeſaid Exceſs, as that begat from CY to that begat from the Fig. CMA turn'd about MK: And by taking away this begat from that, the Reſidue will be the begat from the Triline MKA turn'd about MK. Theſe may be demonſtrated in the ſame Manner as is demonſtrated at Prop. 7. but ſhorter. That begat from MA is to that begat from MCA turn'd about MK, as the Rectang. under MC, ME to the Rectang. under MD, MD; that is, as 121 to 98.

PROP. XII. Fig. 15.

If you let any right Segment of a Fig. of Sines be of the ſame Altitude with the verſed Segment of the ſame, it will be to the whole Fig. as the Baſe of the verſed Segment to the Baſe of the whole Fig. For let the Fig. of Sines be ARB, the Quadrant ABD; the right Segment ILR, the verſed Segment of the ſame Height CNB: I ſay, ILR is to ARB, as CB to AB; for ſince MN and AC are equal, alſo AC and VC; TS will be equal to IL; becauſe IA is equal to MR, therefore IR will be equal to the Arch DS; therefore the Segment IRL is to the whole Fig. ARB, as TD equal to VF, or CB to the whole AC, by Prop. 5.

COROL. I.

Hence the right Trapezium AILB is to the whole ARB, as the right Sine of the Arch equal of Altitude to the Trapezium, *viz.* AI to the whole Sine AB.

COROL. II.

Hence the whole Fig. ARB is to the Segment MRN, as AB to SX; but to the Segment ILR, as AB to TD: Likewiſe to the Trapezium AMNB, As AB to TS; and to the Trapezium AILB, as AB to AT.

PROP. XIII. Fig. 16.

The Solid under the Altitude of the Baſe of the Fig. of Sines proceeding by right Segments under the Altitudes ordinately applied, parallel to the Axis, is Homogene to the Triang. Let ABC be the Fig. of Sines, for Example, the Baſe; let alſo the Fig. of Sines be ABD under the right Angle BAD, and under the ordinate Applicates Parallels BA, OH; let the right Segments, as OHG be equal to EAF: I ſay, this Solid under the Baſe ABC, and the Altitude AD proceeding by the aforeſaid right Segments, of which the Vertexes are terminated at the right Line AD, is Homogene to the Triang. (for Example) ASD. For ABC is to AEF, or HOG, as AD to HD, or AS to HK, by Prop. 12. The ſame may be ſhewn by aſſuming any other right Segment. Therefore the whole Solid ABCD is Homogene to the Triangle ASD.

COROL.

COROL. I.

Hence the aforeſaid Solid ABCD is half the Iſoparallel under the Baſe ABC, and the Altitude AD: For as the Triangle ASD is to the Square AI, ſo is the Solid ABCD to the Iſoparallel ABCD.

COROL. II.

Hence the Solid, which proceeds by Rectangle Parallels under the right Sine, and of the Complement, being applied to AB in the two Figures ABC, ABD is equal to the former Solid. For let there be any Applicate EO, to which another EF approaches at a right Angle OEF; EO is the right Sine of the Arch equal to EB; and EF the right Sine of the Arch equal to EA; therefore 'tis the Sine of the Complement of the firſt Arch; therefore the Rectang. E. F. is under the right Sine, and of the Complement: The ſame may be ſhewn by aſſuming any other right Angle. Therefore that Solid, which proceeds by the Rectangles of this ſort is equal to the Solid ABCD; therefore Subduple of the Iſoparallel under the Baſe ABC, and the Altitude AD.

PROP. XIV. Fig. 17.

The Solid under the Baſe from the generating Quadrant, and from the Altitude of the Axis of the Fig. of Sines, proceeding by Sectors parallel to the Baſe, which ſhall be in the Ratio of Altitudes, is Homogene to the Triang. Let the Quadrant be ALC, the Altitude AF, equal to the Arch of the Quadrant CL, and let any Sector be GID, under the Arch ID, equal to GF, ſince the right Sine of it is GH; I ſay, the aforeſaid Solid is Homogene to the Triang. AFC: Becauſe the Sector ACL is to the Sector GID, as the Arch CL to the Arch DI; that is, as AF equal to the Arch CL to GF, equal to the Arch DI; that is, as AF equal to the Arch CL to GF, equal to the Arch DI, that is, as AC to GO; therefore the Solid, and the Triangle proceed by proportional Elements, therefore they are Homogene.

COROL. I.

Hence the aforeſaid Solid, proceeding by Parallel Sectors, is one Eighth of the Cylinder under the Baſe of the Circle, whoſe Radius is AC, and under the Altitude AF.

COROL. II.

Hence, if a Solid happens of the ſame Altitude AF, proceeding by Rectangle Triangles, under the Baſe and Altitude, from the right Sine and the Complement, it will be $\frac{1}{4}$ of the Iſoparallel Solid, whoſe Baſe is the Fig. of Sines, and the Altitude the ſame of the Fig. of the Baſe; (for Example) let the Triang. be GHD under GH the right Sine of the Arch DI, or KL equal to GF, and under HD, equal to BK, the right Sine of the Arch KC equal to GA. Therefore GHD is under the right Sine, and the Sine Complement: The ſame may be ſhewn in any other aſſumed. And ſince this Solid proceeding by the aforeſaid Triangles, is Subduple of the Solid proceeding by Rectangles under the ſame right Sine, and the Sine of the Complement; for they are Homogenes, and of the ſame Altitude, and every Rectangle is double of its Triangle. And ſince the Solid proceeding by the aforeſaid Rectangle is ſubduple of the Iſoparallel Solid under the Baſe of the Fig. of Sines, and the Altitude the ſame as the Fig. from the Baſe, by Coroll. 1. of Prop. 13. the aforeſaid Solid proceeding by Triangles will be $\frac{1}{4}$ of the aforeſaid Iſoparallel.

COROL. III.

Hence is had the whole Solid ACL, *viz.* the Aggregate from the Procedents by the Sectors, and from the Procedents by the Triangles, for the Procedents by the Triangles is $\frac{1}{4}$ of the aforeſaid Iſoparallel, which is equal to the Cube under the Side AC. Hence the Ratio's may be reduced to Calculation, let the Parallelipiped under the Square of the Baſe AC, and the Altitude AF be 14, it will be to the Cube under the Side AC as 14 to $8\frac{5}{22}$; therefore to the Procedents by Triangles as 14 to $2\frac{10}{11}$; therefore to the Quadrant, or Fourth of the Cylinder, under the Baſe from the Quadrant ALC, and the Altitude AF, as 14 to 11; therefore to the Procedents by the Sectors, as 14 to $5\frac{8}{11}$; therefore to the Aggregate aforeſaid, as 14 to $7\frac{1}{2}$.

COROL. IV.

Hence, if from the aforeſaid Aggregate be taken away the Solid proceeding by parallel Squares AI under the Altitude AC, which is equal to $\frac{2}{3}$ of the Cube, and therefore $5\frac{31}{33}$ of the Reſidue; that which proceeds by Rectangles parallel to GE will be $1\frac{26}{33}$, which if it be added to the Solid AEC, the Aggregate will be $9\frac{17}{33}$.

COROL. V.

Hence, if there be a Parallelipiped under the Altitnde AC, and the Square of the Baſe AF, it will be to the aforeſaid Parallelipiped in the 3d Coroll. as 22 to 14; ſince the Parallelipiped under the Square of AF, and the Altitude AC, is to the Solid, which proceeds by the Squares under AF, BH; that is, under Arches of the ſame Altitude, as 121 to 56, by Prop. 11. Coroll. 1. that is, as 22 to $10\frac{2}{11}$; if from $10\frac{2}{11}$ we take away the Aggregate $9\frac{17}{33}$, as in Coroll. 4. the Reſidue will be $\frac{22}{33}$, that is, the Solid under the Altitude AC, proceeding by Squares, under the Differences of the Sines, and of the Arches, which is to the Solid, proceeding by the Squares under the Sines, as 11, to 98.

COROL. VI.

The Solid AEC proceeds by Rectangles parallels of AE, under the Arches, and right Sines: So AE is under AL the whole Sine, and AF equal to the Arch of the Quadrant. Alſo BD under BH, equal to the Arch KC, and BK the right Sine of the Arch CK: The ſame may be ſhewed in any other aſſumed.

COROL. VII.

Hence if a Solid be made of the Altitude AC, proceeding by Rectangles under the Arches, and under the Compounds from the Arches and the Sines, from which take away a Solid of the ſame Altitude, proceeding by Squares, under the Arches, and which we appoint at Coroll. 5. to be to the Parallelipiped of the ſame Baſe and Altitude AC, as $10\frac{2}{11}$ to 22: Without doubt the Solid remaining of the ſame Altitude AC will proceed by Rectangles under the Arches, and the Sines; therefore 'tis equal to the Solid AEC: Therefore ſince AEC is $7\frac{8}{11}$, the aforeſaid Solid, proceeding by Rectangles under the Arches, and the Compounds from the Sines, will be $17\frac{10}{11}$; to which if we add the Solid proceeding by the Squares of the Sines, likewiſe proceeding by the Rectangles under the Arches and the Sines, the Aggregate will be $31\frac{19}{33}$, and this is the Solid of the Altitude AC proceeding by the Squares under the Compounds from the Arches and the Sines. Alſo the Parallelipiped is had under the Altitude AC, and the Square of the Baſe under the compounded from the Arch of the Quadrant, and the Radius; for it is to the Parallelipiped of the ſame Altitude under the Square of the Baſe AF, as 324 to 121, or as $58\frac{110}{121}$ to 22.

COROL. IV.

Hence, if from the aforeſaid Aggregate be taken away the Solid proceeding by parallel Squares AI under the Altitude AC, which is equal to $\frac{2}{3}$ of the Cube, and therefore 5 $\frac{31}{33}$ of the Reſidue; that which proceeds by Rectangles parallel to GE will be 1 $\frac{26}{33}$, which if it be added to the Solid AEC, the Aggregate will be 9 $\frac{17}{33}$.

COROL. V.

Hence, if there be a Parallelipiped under the Altitude AC, and the Square of the Baſe AF, it will be to the aforeſaid Parallelipiped in the 3d Coroll. as 22 to 14; ſince the Parallelipiped under the Square of AF, and the Altitude AC, is to the Solid, which proceeds by the Squares under AF, BH; that is, under Arches of the ſame Altitude, as 121 to 56, by Prop. 11. Coroll. 1. that is, as 22 to 10 $\frac{2}{11}$; if from 10 $\frac{2}{11}$ we take away the Aggregate 9 $\frac{17}{33}$, as in Coroll. 4. the Reſidue will be $\frac{22}{33}$, that is, the Solid under the Altitude AC, proceeding by Squares, under the Differences of the Sines, and of the Arches, which is to the Solid, proceeding by the Squares under the Sines, as 11, to 98.

COROL. VI.

The Solid AEC proceeds by Rectangles parallels of AE, under the Arches, and right Sines: So AE is under AL the whole Sine, and AF equal to the Arch of the Quadrant. Alſo BD under BH, equal to the Arch KC, and BK the right Sine of the Arch CK: The ſame may be ſhewed in any other aſſumed.

COROL. VII.

Hence if a Solid be made of the Altitude AC, proceeding by Rectangles under the Arches, and under the Compounds from the Arches and the Sines, from which take away a Solid of the ſame Altitude, proceeding by Squares, under the Arches, and which we appoint at Coroll. 5. to be to the Parallelipiped of the ſame Baſe and Altitude AC, as $10\frac{2}{11}$ to 22: Without doubt the Solid remaining of the ſame Altitude AC will proceed by Rectangles under the Arches, and the Sines; therefore 'tis equal to the Solid AEC: Therefore ſince AEC is $7\frac{8}{11}$, the aforeſaid Solid, proceeding by Rectangles under the Arches, and the Compounds from the Sines, will be $17\frac{10}{11}$; to which if we add the Solid proceeding by the Squares of the Sines, likewiſe proceeding by the Rectangles under the Arches and the Sines, the Aggregate will be $31\frac{19}{33}$, and this is the Solid of the Altitude AC proceeding by the Squares under the Compounds from the Arches and the Sines. Alſo the Parallelipiped is had under the Altitude AC, and the Square of the Baſe under the compounded from the Arch of the Quadrant, and the Radius; for it is to the Parallelipiped of the ſame Altitude under the Square of the Baſe AF, as 324 to 121, or as $58\frac{110}{121}$ to 22.

PROP.

I

PROP. XV. Fig. 18.

If the Fig. of Sines be doubled or whole, they will reſemble two Trilines joined by the Baſes, the whole Fig. is Homogene to the Solid begat from the Fig. of Sines turned about the Axis; for ERM will be whole or double of the Fig. of Sines, to which the two Trilines EDR, ADR are alike, being joined by the Baſes in AR: I ſay, the Fig. EARM is Homogene to that begat from the Fig. of Sines (for Example) of EAM turn'd about EA; to wit, as EM to DT, ſo DT to DR; and accepting DF, DC equal; as EM to CQ, the right Sine of the Arch equal to CA, ſo CQ to CX, the verſed Sine of double the Angle in the Quadrant, whoſe Arch is equal to AD, the Half of AE; and as EM to FO, the right Sine of the Arch equal to FA, ſo FO to FZ, of which alſo FY the Difference is ZY equal to CX, by Poſition the 6th: But as EM to DR, ſo that begat from EM to that begat from DT; and as EM to FZ, ſo that begat from EM to that begat from FO; and as EM to CX, ſo that begat from EM to that begat from CQ, for they are begat as the Squares; therefore the Fig. EARM is Homogene to that begat from EAM turn'd about EA.

COROL. I.

Hence EARM is ſubduple of the Rectan. ES, becauſe the aforeſaid begat is Homogene; yea, ſince the Triline ADR is equal to the Triline EDR, the Rectang. EK will be equal to the Fig. EARM, and ſince this is Homogene to the aforeſaid begat, from thence alſo it follows, that the aforeſaid begat is ſubduple of the Cylinder of the ſame Baſe and Altitude.

COROL. II.

Hence the Center of Gravity of the Fig. EARM is had; for ſince the Center of ERM is had, ſuppoſe γ, and of the Triline AER, ſuppoſe δ; draw $\gamma\delta$, and ſo divide it in H, that Hγ be to Hδ, as the Triline ERA to ERM, that is, according to *Archimedes*'s Ratio, as 7 to 4; H will be the Center of Gravity of that you ſeek, as appears from the aforeſaid.

COROL. III.

Hence by uſing Calculation, and by drawing $\gamma\pi$, Hθ; Eπ is to ED, as 4 to 11, and to EA, as 4 to 22; and $\pi\theta$ to θD, as 4 to 7: Therefore let Dπ be 11, and $\theta\pi$ will be 4; Eπ $6\frac{2}{7}$; ED $17\frac{2}{7}$; EA $34\frac{4}{7}$: Therefore Eθ to θA, as $10\frac{2}{7}$ to $24\frac{2}{7}$.

COROL. IV.

The Center of Gravity of the begat from the Fig. of Sines EATM turn'd about AE is θ, for the aforeſaid begat is Homogene of the Fig. EARM; therefore the Center of both will be equally diſtant from the Baſe.

PROP. XVI. Fig. 19.

If we turn about the Axis Plane Figures Homogene to the begat from the Fig. of Sines turned about the Axis, the Ratio is had of the begat to the Cylinder of the ſame Baſe and Altitude; for let EARM be Homogene to the begat from the Fig. of Sines, as aforeſaid, turned about EA; that begat from it, is to that begat from ES in the Ratio compounded from the Ratio of the Fig. EARM to ES, that is, half; and from the Ratio of IH to DR: For I ſuppoſe in H to be the Center of Gravity of the Fig. EARM, for the aforeſaid Solids are as the Moments of each Fig. vibrating in EA, by Poſition 9; and the Moments are in the aforeſaid compounded Ratio, as hath been already ofttimes repeated.

COROL. I.

Hence is found the begat from ERM; now becauſe we have the begat from the Triline EDR, by Prop. 7; therefore equal to this is the begat from the Triline ADR, both being taken away from the begat from EARM being known, the Reſidue will be the begat from ERM. This may alſo be demonſtrated otherwiſe; for the begat from ERM is to the Cylinder of the ſame Baſe and Altitude, as the Iſoparallel under the Baſe ERM, and the Altitude EM, to the Parallelipiped under the Baſe EP, and the Altitude EM, by Poſition 7.

COROL. II.

Hence alſo is had that begat from MPR; to wit, by taking away the begats from ERM, and from EDR known, from the begat from EP known.

PROP. XVII. Fig. 12.

The Center of Gravity is had of that begat from the Triline annex'd to the Fig. of Sines turn'd about the proper Axis. Let the Triline be CBL turned about CL, from it the Center of Gravity of the begat is had from the Centers of the other Solids being known, of which the Axis is common; to wit, we have the Center of the Parallelipiped under the Baſe AE, and the Altitude AB, equal to QY in the Scale. Alſo of the Solid proceeding by the Squares AE, NI, under the Sines, Homogene to that begat from ABC turn'd about AB; let it be T. Alſo of the Iſoparallel under the Baſe ABC, and the Altitude AD of the proceeding by the Rectangles AE, NM, for it is Homogene of the Fig. ABC; let it be X. And as the Difference of each to the Solid proceeding by the Squares AH, NI, ſo TX to XZ in the Scale annex'd to the 12 Fig. Z will be the Center of the aforeſaid Difference; to wit, of the Solid proceeding by Rectangles parallel to GI; alſo in Z is the Center of the other equal Solid proceeding by Rectangles parallel to IS; alſo the Center of the Iſoparallel Solid is had, under the Triline Baſe CLB, and the Altitude CE; for let QY the Scale, be divided in the Middle in θ, and as the foreſaid Iſoparallel under the Baſe CLB, to the Iſoparallel as above under the Baſe ABC, ſo X θ to $\theta\delta$; δ will be the Center of the Iſoparallel under the Baſe CLB; laſtly, let it be as the Solid proceeding by Rectangles parallel IS as above, alſo known, of which the Center is Z, ſo Z δ to $\delta\gamma$, the Center of the Solid proceeding by the Squares parallel BF, IH will be in γ.

But this is Homogene to the begat from the Triline EFK turned about EF; therefore if QY is the Axis of the foreſaid begat, the Center of Gravity of it will be in γ; laſtly, if as $\gamma\theta$ to $\theta\pi$, ſo the begat from CAB turned about CL, to the begat from the Triline CLB; π will be the Center of gravity of the begat from CAB: In like manner the Center of Gravity may be had of the begat from the Triline CLB turned about AB, ſo T θ to $\theta\beta$; β will be the Center ſought.

PROP. XVIII. Fig. 19.

The Center of Gravity is had of that begat from a Fig. Homogene, to the begat from the Fig. of Sines turned about the Axis; for let the foreſaid Homogene Fig. be EARM turned about EA, we have the Center of the begat from both the Trilines EDR, ADR which is in D; likewiſe of the begat from ERM, to wit, of the Homogene to the Iſoparallel Solid under the Baſe ERM, and the Altitude EM by Poſition 7. which is in Z, by granting that the Center known of the Fig. ERM is in X; ſo laſtly, ED will be divided in I, that ZI is to ID, as the begat from both the Trilines EDR, ADR, is to the begat from ERM; I will be the Center of the begat from the Fig. EARM.

PROP. XIX.

If the foreſaid Fig. Homogene to the begat from the Fig. of Sines as aforeſaid, be turned about the Baſe, the Ratio is had of the Solid begat to the Cylinder of the ſame Baſe and Altitude; let them be the ſame as above, and turn the Fig. EARM about EM; let the Center of the Fig. be H, the Solid begat is to the Cylinder in compoſition from the Ratio of ED to EA, and from the Ratio of LH to ED, that is, as the Rectangle under ED, LH, to the Rectangle EA, ED, that is as LH to EA; for the Solids begat are compounded of the Figures, and of the Diſtances of the Centers from the common Axis about which the Revolution is made by Poſition 9.

COROL. I.

Hence alſo are had other begats, to wit, from EDRM; becauſe we have the begat from ERM, by Prop. 11. alſo the begat from the Triline EDR, by Corol. 1. of Prop. 11, therefore the Aggregate from both, therefore the begat from EDRM: Alſo we have the begat from the Triline ADR, for by taking away the begat from ADRM from the begat from EARM, the reſidue will be the begat from ADR; likewiſe we have the begat from SMRA, for by taking away the begat from EARM from the Cylinder begat from ES, the reſidue will give the begat from SMRA, from which if we take away the begat from the Triline MPR known, the reſidue will give the begat from SPRA.

COROL. II.

Alſo we have the begat from EARM turned about MS, for by taking away the begat from SMRA known, from the Cylinder begat from MA, the reſidue will give the begat from EARM.

PROP. XX. Fig. 19.

The Center of Gravity is had of the begat from EARM turned about EM, for we have the Center of the begat from ERM, which is N; alſo of the begat from both Trilines EDR, ADR Homogenes to the Iſoparallel Solid under the Baſe EAR, and the Altitude EA; and ſince the Center of the Triline EAR, is known by Corol. II. of Prop. 10. let it be in Q, draw QV, the Center of the begat from EAR is in V: So laſtly, divide VM in O, that OV be to OM, as the begat from ERM is to the begat from EAR, E will be the Center of Gravity of the begat from EARM, the thing may eaſily be reduced to Calculation.

PROP. XXI. Fig. 20.

We may find the Ratio of the begat from any Segment of a Fig. of Sines; let the Fig. of Sines be DAS, any Segment AQT; let the Fig. DAFL be Homogene to that begat from the Fig. of Sines, as above; the begat from AQT, will be to the begat from DAS, as the Triline TAX to the whole

DAFL; let DC be equal to AT, let alſo the Fig. of Sines be DFV, the Ratio of DFV to DAS being known, which is $\frac{1}{4}$, the Ratio alſo will be known of DAS to AL, which is $\frac{7}{11}$; therefore from theſe is had the Ratio of DAS to DAFL, which is $\frac{1}{2}$ AL, therefore the Ratio is known of DAFL to the Triline DCP; therefore alſo of the begat from DAS turned about DA to the begat from the Segment TAQ.

In another manner, the Ratio being known of the Rectangle DK to DG, equal to the Fig. DAFL, alſo to the Trapezium DPIL, which being taken from DK, the reſidue are the Trilines DEP, LKI; therefore the Ratio is had of the Rectangle DG to the Triline DCP.

COROL. I.

Hence by tranſlating the Segment TAQ into DCZ, and by making the Fig. of Sines EMK by producing the Axis FM, and then dividing VN in the Middle, draw RY parallel to LS; if the two Trilines LKI, DCE librate in YR, it will be equal weight; For as IK to DE, ſo any Parallel of IK to any Parallel of DE equidiſtant from RY; but as DCE to LKI, ſo the begat from TAQ or DCZ to the begat from Sγδ, to wit, as the begat from DZ is equal to the begat from δγ, ſo the begat from any Parallel to DZ, is equal to the begat from the Parallel to γδ equidiſtant from YR: Therefore the Moments of the Weights of thoſe begat from DCZ, and from Sγδ in YR are equal; likewiſe the begat from CδSZ, to wit, as PI is equal to EH, and any other Parallel to PI, equal to another Parallel equidiſtant from OR, ſo the begat from Cδ equal to the begat from SZ; And any other begat from a Parallel to Cδ equal to the begat from another equidiſtant from YR.

COROL. II.

Hence is had the Diſtance of the Center of Gravity of the Segment TAQ from the Axis TA, for ſince the Ratio is had of the begat from TAQ, to the begat from the Rectangle under TA, TQ; alſo the Ratio of the Segment TAQ to the foreſaid Rectangle, ſince thoſe begat are in compoſition, or are compounded from the Ratios of the Figures, and of the Diſtances of the Center of each from the common Axis TA: Laſtly, ſince we have the Diſtance of the Center of the aforeſaid Rectangle, which is $\frac{1}{2}$ TQ, from thence we have the other Diſtance, *viz*, of the Center of TAQ from the ſame TA; which that it may appear more clear, ſee Fig. 21. let the Ratio of thoſe begatters be AB, AD, let the Ratio of the Figures be AB, AC, let BC be a Diſtance known, to wit, of the Center of the Rectangle common from the Axis; let the Rectangles be AE, AG, AB, draw KC, FIL; FG will be the Diſtance ſought; and the Rectangles are AE, LG, as AB, AD, and in compoſition from the Ratios of AB to AC, and of BE to FG.

COROL. III. Fig. 20.

Hence is had the Diſtance of the Center of Gravity of the Trapezium DAQZ from the Axis AD; becauſe we know the Diſtance of the Center of the Rectangle

angle DQ, alſo of the Segment TAQ; therefore we know the Aggregates from both; likewiſe we have the Diſtance of the Center of Gravity of the Trapezium DTQS from the Axis DA, ſince we have the Diſtances of the whole DAS, and of the Part TAQ, of the other Part alſo may be had.

COROL. IV.

Hence is had the Diſtance of the Center of Gravity of the verſed Segment ZQS from the ſame DT, to wit, ſince we have the Diſtances of the whole DTQS, and of the other Part, to wit, of the Segment ZQS, therefore alſo the Diſtance of the Center of the ſame ZQS from the Axis ZQ; therefore we have the begat from the Segment ZQS turned about ZQ, for it is to the Cylinder of the ſame Baſe and Altitude in the compounded Ratio from ZQS to ZT, and from the Ratio of the Diſtance of the Center ZQS from ZQ to the half of ZD.

COROL. V.

Hence from deſcribing the Quadrant DST ſince the Perpendicular is had falling on ZS from the Center of Gravity of the Segment ZQS, likewiſe the Perpendicular falling on ZS from the Center of Gravity, Z β S is had alſo the Perpendicular falling on the ſame DS, from the remaining Fig. SQT.

PROP. XXII. Fig. 22.

The Homogene Fig. begat from the Fig. of Sines turned about the Axis, is equal to the Circle whoſe

Diameter is the Baſe of the Fig. Let the foreſaid Homogene Fig. be ASIH, let the Semicircle be AKH; ſince this is equal to the Rectangle AI, and therefore the whole Circle equal to the Rectangle AL, to which ASIH is equal, this will be equal to the Circle.

COROL. I.

Any Parallels being drawn to AS, as CT, FV, XZ: As AS is equal to the Arch AKH, ſo CR to the Arch ONH; and XM to the Arch NH; from XM appears from conſtruction of the Line of Sines IMH; for ſince FX is the Sine of the Arch KN, *viz.* equal to MZ; MX will be equal to NH, to wit, FI is ſuppoſed equal to the Arch KH; and ſince RT is equal to the Arch ω A, CR will be equal to the Arch ω KH; the ſame may be ſhewn of any other aſſumed.

COROL. II.

Hence by taking away the Semicircle AKH, the remaining Fig. is equal to the Semicircle; and the Fig. of Sines ASZH is equal to the Quadrate under AH. Laſtly the Leaf HISZH is equal to the Difference of the foreſaid Square, and of the Circle under the Diameter AH.

COROL. III.

If from the Center of Gravity of the Fig. ASIH you let fall a Perpendicular on AH, ſuppoſe on D, AD will be to DH, as 3 to 5, to wit, from the

Center AIH it will fall on F, likewiſe from the Center of the Triline API it will fall on B; let FA be 7, AB will be $2\frac{3}{16}$; ſo BF being divided in D, that BD be to DF as API to AIF, that is, as 4 to 7, or as 16 to 28; DF will be $1\frac{3}{4}$, therefore DA $5\frac{1}{4}$; DH $8\frac{3}{4}$, therefore AD to DH, as $5\frac{3}{4}$ to $\frac{3}{8}$, $8\frac{3}{8}$, or as 21 to 35, that is as 3 to 5, hence if it be divided into 8 equal Parts, AD will be 3, and DH will be 5 of the ſame Parts.

COROL. IV.

If from the Center of the Fig. of Sines FIH, we let fall a Perpendicular on GI, FG will be $\frac{1}{4}$ of the whole FI; and as FH to FG, ſo the Square under FH, to the Semiquadrant of the Circle under the Radius FH; therefore by hanging from F the Fig. FIH in the Perpendicular IF to the Arm of the Beam or Ballance AF, the Semiquadrant in A will make equal Weight, or equal Moment; for ſince the Moments are compounded of the Quantities and of the Diſtances, the Moment in G is to the Moment in A, as the Rectangle under FH, FG, to the Rectangle under FG, FH; to wit, in G is the Quantity FH, the Diſtance GF; but in A the Quantity FG, the Diſtance FA or FH, therefore both Moments are equal.

COROL. V.

If Hδ be equal to AS, alſo FI equal to FY, and you make Y$\beta\delta$ the Line of Sines, the Fig. IHδY will be Equal and Homogene to the Rectangle LY; for as Hδ is equal to Lθ, ſo BM is equal to πZ; the ſame may be ſhewn of any other aſſumed; therefore the foreſaid Fig. is Homogene to the Rectangle LY, therefore equal to the ſame, ſince 'tis of equal Altitude HF, and of equal Baſe H θ, therefore the foreſaid Fig. is equal to the Circle under the Radius FH, therefore alſo of the Fig. ASIH.

COROL. VI.

If the foreſaid Fig. be hung from F in the Perpendicular IF, on the End or Arm of the Ballance AF, the Semicircle AKH in A will make equal Weight, for divide HF in the Middle in X, and draw XZ β, it will go or paſs through the Center of the Fig. therefore the Moment in X is equal to the Moment in A, ſince the Ratio of the Weights is $\frac{1}{2}$, and of the Diſtances $\frac{2}{1}$ therefore compounded of $\frac{2}{2}$, therefore the Moments are equal; the ſame will be done if we hang the Perpendicular LH on the Arm of the Beam or Ballance H γ equal to HF.

COROL. VII.

Let there be the ſame Perpendicular LH, and the Arm of the Ballance H γ, for the Fig. of Sines FIH being hung, the Square under HF leſs $\frac{1}{8}$ of the Circle will make Equilibrium, as is manifeſt; but for the other Fig. FY δ H, they will equiponderate $\frac{1}{8}$ of the ſame Circle, leſs by the Square under HF.

COROL.

COROL. VIII.

But if the Perpendicular be AP, the Arm of the Ballance AE equal to AF, for the Fig. ASIF being hung; they will make equal Weight $\frac{5}{8}$ of the ſame Circle, leſs by the Square under AF; becauſe 'tis equal to the Fig. H ♂ YF already hung to the Perpendicular LH, and of the ſame Poſition; and for the Fig. of Sines FIH hung to the Perpendicular AP, it will make Equilibrium with $\frac{1}{8}$ of the ſame Circle, more by the Square under AF; therefore from the Ratio of the whole Fig. ASIH being hung, they will make Equilibrium $\frac{6}{8}$ of the foreſaid Circle, becauſe if we add $\frac{5}{8}$, leſs by the Square of $\frac{1}{8}$ more by the Square, the Sum will be $\frac{5}{8}$; therefore as 8 to 6, or as 4 to 3, ſo EA to AD, for from the Center of the Fig. ASIH the Perpendicular will fall in D; but AF is equal to AE, therefore AD is to DH, as 3 to 5, as is already ſhewn above.

COROL. IX.

If we ſuppoſe the Circle whole, the ſame Center A, ſince the Diſtances AF, EF are equal according to Equilibrium, the ſame thing will happen in the Moment E, therefore $\frac{4}{8}$ of the Circle; therefore thoſe of the Moment E compound $\frac{10}{8}$ of the Circle, but the oppoſite equal Moment compounds $\frac{12}{8}$ of the Circle; therefore the Diſtances are in the ſame Ratio by changing, therefore as 12 to 10, or 6 to 5, ſo EA to another (for Example) to AO, in O will fall the Perpendicular from the Center of the Fig.

And ſince A F is equal to AE, let AH be 12, AO will be 5, and OH will be 7.

COROL. X.

Hence if the Fig. ASIH be turned about AS, that begat is to the Cylinder of the ſame Baſe and Altitude, as 3 to 8, for it is compounded from the Ratio of the Fig. ASIH to the Rectangle under AS, AH which is $\frac{1}{2}$, and from the Ratio of AD to AF which is $\frac{3}{4}$, therefore 'tis compounded of $\frac{3}{8}$.

COROL. XI.

If we ſuppoſe an whole Circle from the Center A, and the Fig. turned about as above, or before the Ratio of the begat to that of the Cylinder is had; to wit, being compounded from the Ratio of the Fig. ASIH more by the Semicircle, to the Rectangle under AH and AS more by AF; and from the Ratio of AO to AF.

COROL. XII.

If from the Fig. ASIH be taken away the Semicircle, that it remain in Equilibrium, the Remainers as ſtanding above, the Semicircle ought to be taken away, that is, $\frac{4}{8}$ to the Moment E; therefore there remains $\frac{2}{8}$ for the ſame Moment E at Equilibrium; therefore ſince the Ratio of Weights is $\frac{1}{2}$, the Ratio of Diſtances will be $\frac{2}{1}$; therefore if AC be ſubduple of AE, or AF, the Perpendicular will fall in C, from the Center of Gravity of the Fig. ASIH, by taking away the Semicircle AKH.

COROL.

COROL. XIII.

If we turn about the Fig. ASIH, by taking away the Semicircle AKH, that begat is to the Cylinder, as 1 to 8, *viz.* in Compoſition from the Ratio of the Fig. to the Rectangle under AS, AH, which is $\frac{1}{4}$; and from the Ratio of AC to AE which is $\frac{1}{2}$; therefore the compounded is $\frac{1}{2}$; Hence that begat from the whole ASIH is triple of the foreſaid begat; and the begat from the Semicircle AKH double; hence by aſſuming the whole Circle, that begat from the Fig. is to the Cylinder under the ſame Baſe and Altitude AS, as 5 to 8, to wit, the begat from the Semicircle is to the aforeſaid Cylinder, as 2 to 8; therefore that begat from the Circle, as 4 to 8, that begat from the Reſidue, as 1 to 8, therefore that begat from the whole, as 5 to 8.

COROL. XIV.

The Center of the Fig. ASIH is had, for by drawing from the Center of the Fig. AIH known, the right Line to the Center of the Triline AIS known, and that will be the Center of the whole Fig. in which the foreſaid right Line will cut the Perpendicular by falling in D, ſuppoſe ω.

COROL. XV.

Hence is had that begat from the Fig. ASIH turned about AH, for it is to the Cylinder of the ſame Altitude and Baſe, as the Rectangle under AF, D ω, to the Rectangle under AH, AP, to wit, in the Compoſition from the Ratio of the Fig. to the Rectangle and of the Diſtance D ω, to the Diſtance AP.

Therefore that I may compriſe the whole in brief, let BEFC be like to the former, alſo BESA, they will reſemble BGHC, HGIA, the two Semicircles are under the Diameters AB, BC; likewiſe the Fig. of Sines BFC, BSA; let S δ be equal to ML, and the other in like manner to the other Sines, bein terminated with the Line A δ E; laſtly let the Fig. of Sines be AIB, the Semicircle BXA, the Parallels OR, ZV, &c. theſe being granted or ſuppoſed, if about EG we turn BEFC, that begat is to the Cylinder of the ſame Baſe and Altitude, as 3 to 8; if by taking away the Semicircle the Remainder of the Fig. be turned about, that begat is to the Cylinder, as 1 to 8; if we turn about the Fig. of the Semichord BGIAL, that begat is to the foreſaid Cylinder, as 5 to 8; in like manner is had that begat from BFC, which is to the Cylinder of the ſame Altitude, as the Square of the Radius to the Semicircle, alſo the begat from the Triline BFC.

COROL. XVI.

If we turn the Fig. BESA about the Tangent in A, that begat is to the Cylinder of the ſame Baſe and Altitude, as 5 to 8, *viz.* in Compoſition of the Figures $\frac{1}{2}$, and of the Diſtances from the Center $\frac{5}{4}$: If we turn about the whole AEC, that begat will be $\frac{1}{2}$ of the Cylinder under the ſame Baſe and Altitude, for the Ratio of Diſtances is of Equality;

therefore thoſe begat are as the Fig. If we ſubtract the Semicircle, and the remaining Fig. BESA be turned about, that begat will be to the Cylinder as 3 to 8, to wit, compounded from the Ratio of the Figures $\frac{1}{4}$, and of the Diſtances $\frac{3}{2}$; if we ſubtract each Semicircle, and turn about the Reſidue of the whole Fig. AEC, which is like to a Lilly, that begat will be to the Cylinder as 1 to 4, which is the Ratio or Proportion of the Figures, for they are equal of diſtance; if we turn about the Fig. of the half Chord, it will be to the Cylinder under the Altitude BG, and of the Baſe begat from AB, as 7 to 8: If laſtly the whole Heart, that begat will be to the Cylinder under the Altitude BG, and of the Baſe begat from AC, as 6 to 8, *viz.* the begat from the Fig. AEC, is to the aforeſaid Cylinder as 4 to 8, that begat from the Lilly, as 2 to 8, therefore that begat from both Semicircles as 2 to 8, therefore that begat from the Heart, that is from the Fig. together and the double Semicircle, is as 6 to 8; hence that begat from the Semi-heart turned about BG, is equal to that begat from BCSA about the Tangent A; alſo that begat from the Fig. BCSA about BE, is equal to that begat from the Reſidue of the ſame Fig. from which the Semicircle is taken away, about the Tangent A.

COROL. XVII.

If the Revolution be made about AC, ſince the Ratio of the Figures, and of the Diſtances is had from the Center, the Ratio of thoſe begat is had which is compounded from both, and we have the Center as well of the Fig. AEC, as of the Lilly, as appears: Alſo from the turning abont being made about the

Tangent is had that begat from the Fig. BESA; for by taking away from the Cylinder the begat from the ſame turn'd about BA, the Remainer will give that begat from the ſame Fig. about the Tangent E. Likewiſe is had that begat from the remaining Fig. BESA from the ſubtracted Semicircle; to wit, is had that begat from the Semicircle ALB being turned about the Tangent E, for it is half of the Cylinder under the Altitude BA, and of the Baſe begat from EB, leſs the Sphere under the Diameter AB; but by taking away the begat from the Semicircle is had that begat from the Remainer, therefore that begat from the Lilly; therefore alſo the begat from the Triline BSE, likewiſe that begat from the Fig. ESAIG, which is half of the Cylinder, for ſince the Diſtances are equal, thoſe begat are as the Figures.

COROL. XVIII.

If we turn about the Tangent G the Fig. of the half Chord, the begat from it is had; to wit, the begat from the Semicircle ALB is had; and as the Parallelipiped under the Square of GB, and the Altitude BA to the Cylinder under the Altitude GB, and of the Baſe from the Circle AXBL, more by the ſolid Homogene to the Hemiſphere, proceeding by the Squares of the Sines under the Altitude AB, ſo that begat from the Rectangle GA turned about G, to that begat from the Semicircle ALB; therefore by taking away the begat from BESA turned about BA, from the begat from the Rectangle GA turned about G, the Remainer ſince being begat from a Semicircle, will give that begat from the half Heart about G, hence are had thoſe begat from the whole Circle AXBL, from the whole Figure of the Heart, from the Lilly, from the double Lilly by taking away the Circle on both Sides; all theſe may be reduc'd to Calculation, ſuppoſing *Archimedes* his Ratio.

COROL. XIX.

The Center of Gravity of that begat from BESA, turned about BE, may eaſily be found; to wit, the Center of that begat from the Triline BSE is in D, but of that begat from BSA 'tis equally diſtant from AC, as the Center of the Fig. BSA; therefore the Centers of the Begats ABSA and BSE are had in EB; therefore is had the Center of the begat from BXA, therefore from the common of the begat from BXAS; therefore of the begat BESAX, alſo of the begat from the Reſidue BESA, to which the taken away is BLA; therefore alſo the Center of that begat from the Figure of the Heart.

COROL. XX.

The Fig. contained by the Curve Line A♂E, and by the Curve Line ASE is Homogene to the Semicircle ALB, and equal to the ſame, as appears; alſo that begat from BSEAX and BE♂A are equal; likewiſe the Reſidue of the Fig. BE♂A, by taking away the Semicircle BLA, is equal to the Fig. BESA; alſo that begat from both are equal; likewiſe that begat from MSA about MA is had, which is Homogene to the Solid of that which proceeds by the Squares of the Arches known; alſo that begat from M ♂ A about MA, which is Homogene to the Solid which proceeds by the Squares of the Compounds from the Arches and Sines alſo known by Corol. 7. Prop. 14. But the Solid proceeding by the Squares of the compounded from the Arches and Sines IL, TV, is equal to the proceedent by the Squares M ♂, and of the Parallels terminated at the Curve Line A ♂ E; alſo equal to the proceedent by the Squares of IL and QR, &c. Laſtly, if from the Parallelipiped under the Square of GB, and the Altitude BM, be taken away twice, the Iſoparallel under the Baſe from the Triline LSB, leſs by the ſolid proceeding by the Squares under LS, and the other Differences of the Arches and Sines under the Altitude BG, being known by Corol. 5. Prop. 14. the Reſidue will give the Solid under the Altitude BM, proceeding by the Squares GB or IL equal to the proceedent by the Squares BE, M ♂ under the ſame Altitude; hence is had the whole ſolid under the Altitude AB, proceeding by the Squares BE, M ♂, &c. and this is to the Parallelipiped under the Altitude AB, and of the Baſe from the Square of BE, as that begat from the Fig. BE ♂ A turned about BA to the Cylinder of the ſame Height and Baſe. Theſe Things have been ſo often demonſtrated that it were a Shame to repeat them.

PROP. XXIII. Fig. 24.

The Semicycloid terminates the compounded from the Arches and Sines ordinately applied to the Diameter of the Circle; that theſe Things may be underſtood with thoſe which are ſaid and demonſtrated,

ſtrated, ſomewhat of new Conſtruction is to be uſed; let CB be the Diameter of the Semicircle BDC inſiſting at Rectangles to CG, and draw how many ſoever Parallels OT, AI, KX, let LN be equal to the Arch LB; alſo DF equal to the Arch DLB, OS to the Arch PLB; Laſtly, CG equal to the Arch of the Semicircle CDB, and through the extream Points BNFG, ſuppoſe the Curve drawn; moreover, ſuppoſe the Semicircle CDB, ſo moved in CG by a right Motion of the Center through AI, and by the Motion of the Orb about A, that each Motion be equal and made together: Furthermore aſſume LM equal to KL; DE equal to AD; PQ equal to OP; and the ſame may be done in any aſſumed, and through the extream Points BMEQC draw the Curve; Laſtly, Let the Curve CRFH be like to the former BNFG, and the Fig. of Sines IYH; theſe being poſited, I will demonſtrate the Line BNFG to be the Semicycloid, that is the Curve Line deſcribed from the Point B, in the aforeſaid Motion of the Semicircle; by Definition 2. move B by the Motion of the Orb through the Arch BD, it will recede from BC by the Space AD, that is the Sine of the Arch DB; and together or at the ſame time let the Motion of the Center move with an equal Motion to the Orb, it will recede from BC by the Space DF, equal to the Arch DB, for equal Spaces ſuppoſe equal Motions in equal Time, therefore from both Motions together the Point B will recede from BC, by the Space AF; and ſince the Arch BD paſſing over is in the Line AI, it will be in F: In the ſame Manner it may be proved by the Arch BL paſſing over, that the Point B is in N; from the Arch PLB paſſing over 'tis in

S; from the Arch CDB paſſing over, 'tis in G; Laſtly, From any other Arch paſſing over, to terminate the Applicate.

And that all Ambiguity may be taken away, I ſhall ſubjoin ſome Definitions of Words; the Curve Line BNG is called a Semicycloid; the Fig. of half the Cycloid CBFG; the Axis BC; the Baſe CG; the generating or begetting Circle BDC; the ordinate Applicate AF, the right Segment KBN; the verſed Segment γ FG; the right Trapezium CKNG; the verſed Trapezium CBFγ.

COROL. I.

Any Segment ordinately applyed, intercepted between the Arch of the Semicircle CDB, and the Curve BFG, is equal to the Arch cut of (for Example) DF is equal to the Arch cut of DB; LN equal to the Arch LB; PS equal to the Arch PLB.

COROL. II.

Hence the Curve Line Aδ E, in Fig. 23. is an half Cycloid; and BAδ E the Fig. of the Cycloid.

COROL. III.

Hence the Semicycloid above, being rightly determined or limited, as well by the Motion of the Circle or Wheel, or of the Circle on a Plane, ſo that the Motion of the Orb, and of the Center are equal, as by the ordinate Applicates to the Diameter of the Circle, compounded from the Sines and the Arches.

COROL. IV. Fig. 24.

The Fig. BEC is Homogene to the Semicircle BDC; for as AD is to AE, ſo KL to KM, *&c.* 'tis called the Terminate of the double of the Sines, and it is an Ellipſis, as appears; likewiſe the Fig. contained by the Curves BCD and BEC is Homogene on both Sides.

COROL. V.

The Fig. contained by the Curves BEC and BFG is Homogene to the Triline BHG, to wit, EF is equal to FI, for EF is the Difference of the Sine AD, and the Arch DB; alſo FI more DA equals the Arch DC, therefore FI is equal to EF, likewiſe ST more OP equals the Arch PC; therefore by ſubtracting from OT, the Arch PC more PO, the Reſidue is QS; and aſſuming KB equal to OC, by ſubtracting from KX, the Arch LB equal to PC, or LN more KL equal to CP, that is, by ſubtracting the whole KN, the Reſidue NX will be equal to QS, the ſame may be ſhewn in any other aſſumed; therefore they are Homogene Figures; it may be ſhewn otherwiſe thus, ſince OP more ST is equal to the Arch PC, and conſequently of the right Line PR, and let PQ be equal to PO, QR will be equal to ST; in like manner MN is equal to VX, therefore MV equal to NX; therefore the Triline GHB Homogene of the Fig. contained by the Curves CEB, and CFB,

and

and the right Line HB, therefore alſo equal by Poſiton 1. hence alſo equal to the Semicircle CDB, is the Fig. contained by the Curve Lines CDB, CEB.

COROL. VI.

Hence the Triline GIF is equal to the Triline IFH, alſo the Trapezium CEFG is equal to the Trapezium CFIG.

COROL. VII.

The Triline DEB is Homogene to the Fig. of Sines; for as DF is equal to IY, ſo LN is equal to XZ, *viz.* to the Arch LB; I ſay the ſame by any other aſſumed, therefore the Triline is Homogene to the Fig. therefore alſo equal.

PROP. XXIV.

The Fig. of the Cycloid is equal to three generating Circles, or, which is the ſame, the Fig. of the half Cycloid is equal to three half Circles; this Propoſition ſome Men have already demonſtrated, as *Torricellius* in his Appendix concerning the Dimenſion of a Cycloid; I ſhall not retrude from his Invention, but after my Manner ſhall explain the Thing by a four-fold Demonſtration.

1. The Rectangle CI is equal to the Circle of the Generator or Begetter by Corol. 2. Poſition 2; the Triline GIF is equal to the Triline IFH by Corol. 6. of Prop. 23; ABE is equal to the Semicircle by Corol. 5. therefore ABE more CI, or,

which is the ſame, the Fig. of the Semicycloid CBG is equal to three Semicircles.

2. CH is equal to two Circles; CEB equal to the Circle, therefore the Reſidue is equal to the Circle; but the half of the Reſidue is the Triline GHB by Corol. 5. therefore equal to the half Circle; therefore the other half contained by the Curve Lines BEC, BFG, and the right Line CG, is equal to the half Circle, therefore the whole Fig. CBG is equal to three Semicircles.

3. The Triline DFB is equal to the Fig. of Sines IYH by Corol. 7. of Prop. 23. therefore to the Square under AB, by Corol. 1. Prop. 3. the Trapezium CAFG is equal to the Circle leſs by the Triline GIF; likewiſe the Triline EFB is equal to the Difference of the Quadrant, and of the Quadrate; therefore ABF is equal to the Semicircle more by the foreſaid Difference; the Trapezium is equal to the Circle leſs by the ſame Difference; therefore both together, that is, the whole Fig. CBG is equal to three Semicircles.

4. The Fig. of the half Heart in Fig. 23 is equal to three Semicircles, as hath been ſhewn at large in Prop. 22. and this is equal to the Fig. CBG, becauſe Homogene; for both proceed by Applicates compounded from the Arches and Sines, by Corol. 2. therefore the Fig. of the half Cycloid is equal to three Semicircles, therefore the whole Fig. of the Cycloid is equal to three generating Circles.

COROL. I. Fig. 24.

The Section contained by the right Line BG, and the Curve Line BFG, is equal to the Semicircle, becauſe the Triangle CBG is equal to the Circle.

COROL. II.

The Triline ♂ FB is equal to the Quadrate under AB; becauſe the Triangle A B ♂ is equal to the Quadrant, and the Fig. ABF equal to the Quadrant, more by the Quadrate.

COROL. III.

The Triline LBN more by the Trapezium CPSG is equal to the Rectangle KH, becauſe PQC is equal to KBL; QRC is equal to XHV; Laſtly, The Trapezium CRSG is equal to the Trapezium BNVH: The ſame may be ſhewn if any other be aſſumed: And KH is to the Circle as BK to BA; hence the Trapezium DLNF more DPSF is to the Circle, as AK to AB; hence the Cylinder under the Altitude KB, and of the Baſe from the Circle under the Radius AB, is equal to the Iſoparallel under the Baſe KB and the Altitude AB, for they are compounded from the Ratio of the Baſes which is BA to BK, and of the Altitudes, which is BK to BA.

COROL. IV.

♂ EB is equal to the Quadrant, becauſe the Triline EFB is equal to the Difference of the Quadrant and the Quadrate; alſo ♂ FG is equal to twice the Segment of the Circle contained by the Arch of the Quadrant, and the Subtenſe ED, likewiſe the Curve Line BFG will divide in the Middle the Triangle BGH; alſo the Triline CFG will contain four times tne aforeſaid Segment, therefore 'tis double of the Triline HFG.

PROP. XXV.

The Ratio is had of any given right Segment of the Fig. of the Cycloid to the whole Fig. Let firſt of all the Segment be ABF, it is to CBG, as the Quadrant more by the Square under AB to three Semicircles, that is, according to *Archimedes*'s Ratio, as 25 to 66. Moreover, let another Segment be KBN, the Ratio is known of LBN, or XZβ to the Square under AB, by Prop. 6. therefore to the Quadrant; alſo the Ratio will be known of the $\frac{1}{2}$ Segment KBL to the Quadrant; for it is equal to the Rectangle under $\frac{1}{2}$ LV and AB, leſs by the Triang. BKL; therefore the Ratio is had of the Segment KBN to the Quadrant; therefore to the whole CBG, which contains 6 Quadrants: In the ſame manner is had the Ratio of the Segment OBS, becauſe CT is had, alſo GTS equal to MNB, which being taken from CT, and the Reſidue being taken from the Whole CBG, the Reſidue will be OBS: Or ſhorter thus, Since EMNF is equal to FITS, from the given OI, ABE, and MNB, is had OBS equal to the aforeſaid taken together.

PROP. XXVI.

The Center of Gravity of the Cycloid may be had; let the Fig. of half the Cycloid be CBG, by taking away the Semicircle CDB, the Remainer is an Homogene Fig. Homogene to that begat from the Fig. of Sines turned about the Axis, ſince each Fig. proceeds by Applicates equal to the Arches; but the Perpendicular falling from the Center of the aforeſaid Homogene Fig. will ſo divide the Baſe that the Segments ſhall be in Ratio $\frac{3}{5}$, by Corol. 3. of Prop. 22. Therefore from the Center of the Fig. contained by the Curves BDC, BFG, and the right Line CG, the Perpendicular falling on the Axis BG, it will cut it ſo, that the verſed Segment B is to the verſed Segment C, as 5 to 3. But if the aforeſaid Homogene Fig. reſemble a Semicircle in that Manner which is ſaid above, the Perpendicular falling from the Center on the Axis will ſo cut it that the Segments are in Ratio $\frac{5}{7}$, by Coroll. 9. of Prop. 22. but the Fig. added to the Semicircle is Homogene of the Fig. CBG, by Coroll. 2. of Prop. 23. Therefore a Perpendicular falling from the Center of the Fig. CBG on the Axis, will ſo cut it, that the verſed Segment B will be to the other verſed C, as 7 to 5. Hence if CB be of 12 equal Parts, by aſſuming or taking from the Vertex B 7 of them, there will be the Center of the Fig. of the Cycloid.

COROL. I.

Hence the Center of Gravity of any right Segment of a Fig. of a Cycloid is had; for Example, let it be ABF: The Center of Gravity of the Fig. IYH is had by Prop. 10. therefore by drawing from this Center the Perpendicular on BA, it will paſs through the Center of the Homogene Triline DFB, by Poſition 8. In like manner is had the Center of the Quadrant ADB, and by drawing from it a Perpendicular on AB in the End it will ſo divide the Segment of the Axis intercepted by the Perpendiculars, that the verſed Part B will be to the other verſed A, as the Quadrate to the Quadrant, there will be the Center of the Segment BAF. And that we may ſhew ſomewhat of Calculation, the Center of ABD is eaſily had, for ſince the Ratio of the Figures, that is, of the Quadrate under AD, and of the Quadrant ADB is $\frac{14}{11}$: Likewiſe the Ratio of the Solids of thoſe begat being made by the Revolution about AD $\frac{3}{2}$, and let the other of the Diſtances be half of AB, (for Example) 7, the other Diſtance will be $5\frac{31}{33}$, this is the Diſtance of the Center of Gravity of the Quadrant from AD. Moreover, the Center of the Triline DFB will be diſtant from AI, $\frac{1}{4}$ of the whole DF, which ſince it is 22 by ſuppoſing that AB is 14, therefore the Diſtance will be of Parts $5\frac{1}{2}$,

Let another Segment be KBN, we have the Diſtance of the Center of Gravity of the Segment XZH from the Axis XZ, by Coroll. 4. of Prop. 21. therefore alſo of the Homogene Fig. LNB. Likewiſe the Diſtance of the Center of the Segment KLB from the ſame KL will be known; therefore if it be made as above, we ſhall have the Diſtance of the Center of the whole KBN from the ſame KN. Laſtly, Let the Segment be OBS, the Center of ABF will be had, likewiſe of KBN, therefore alſo of the other Part AKNF, alſo of ABE, and of KBM, therefore of EFB, likewiſe of EMNF, therefore alſo of FITS, likewiſe of OI, therefore of OAFS, therefore alſo of the whole Segment OBS: And that theſe may be reduced to Calculation, ſuppoſe the Quadratures of the Circle.

COROL. II.

Hence alſo the Center of Gravity is had of the other Parts of the Fig. of the Cycloid in the Axis BC, for Example, of MNB; for ſince the Center of the whole Segment KBN is had, alſo of KBM, we ſhall have alſo the Center of the other Part MNB, likewiſe of STG, and of XHV, which are Homogene Figures to the Triline MNB: We have alſo the Center of the Triline CFG, ſince we have as well the Center of the whole CBG, as of the Part CEB, alſo of COSG. Alſo of the Fig. contained by the Curve Lines BEC, BFG, and the right Line CG being had; to wit, from the Center of the whole CBG, and of the Part CEB, therefore alſo the Triline GHB, alſo of CEFG, likewiſe of FIHB, alſo of the Section contained by the right Line BG, and the Curve Line BFG, and laſtly of any Trapezium.

PROP. XXVII.

If we turn about the Fig. of the Cycloid about the Baſe, that begat is to the Cylinder of the ſame Altitude, and the Baſe of the Circle under the Radius of the Axis of the equal Fig. as 5 to 8. Let the Fig. of half the Cycloid be CBG turned about the Baſe CG, that begat is to the Cylinder begat from the Rectang. CH, as 5 to 8; to wit, in a compounded Ratio from the Ratio of the Figures $\frac{3}{4}$, and of the Diſtances $\frac{5}{6}$, which is $\frac{15}{24}$ or $\frac{5}{8}$, *viz.* the Diſtance of the Center of the Fig. CBG from CG, is to CA, as 5 to 6, by Prop. 26: Add that the

Fig. CBG is Homogene to the Fig. of $\frac{1}{2}$ the Heart, and that begat from it to that begat from theſe is Homogene; and that begat from the Fig. of the $\frac{1}{2}$ Heart to the Cylinder, as 5 to 8, by Coroll 15. of Prop. 22. therefore alſo that begat from CBG to the Cylinder begat from CH, as 5 to 8; the ſame may be ſaid concerning that begat from the whole Fig. of the Cycloid to the Cylinder under the Baſe from the Circle begat from CB, and from the Altitude of double CG.

COROL. IV.

Hence that begat from the Triline GHB is to that begat from the Fig. CBG, as 3 to 5; but to the Cylinder as 3 to 8.

PROP. XXVIII.

That begat from the Fig. of half the Cycloid turned about the Baſe, by taking away the generating Semicircle, is to the aforeſaid Cylinder as 3 to 8; for this is an Homogene of the Homogene Fig. begat from the Fig. of Sines turned about the Axis, therefore thoſe begat from both are Homogenes: But that begat from the other is to that Cylinder as 3 to 8, by Coroll. 15. of Prop. 22. therefore alſo that begat from the other to its Cylinder, as 3 to 8; for let the aforeſaid Fig. contained by the Curves BDC, BFG, and the right Line CG be turned about CG, that begat is to the Cylinder, compounded from the Ratio of the Diſtances, which is $\frac{3}{4}$, by Prop. 26. and Coroll. 3. of Prop. 22. and from the Ratio of Figures, which is $\frac{1}{2}$; but from

theſe

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theſe the Compound is $\frac{3}{8}$, therefore the aforeſaid begat is to the Cylinder as 3 to 8.

COROL. I.

Hence that begat from the aforeſaid Fig. is equal to that begat from the Triline GHB, which is to the Cylinder as 3 to 8, by Coroll. of Prop. 27.

COROL. II.

Hence that begat from the Semicircle CDB is to that begat from the aforeſaid Fig. as 2 to 3, and to the Cylinder as 1 to 4.

COROL. III.

And ſince that begat from CEB is double of that begat from CDB, for thoſe begat are as the Figures; that begat from the Fig. contained by the Curve Lines BEC, BFG, is ſubduple of that begat from the Semicircle CDB; for that begat from the Fig. contained by the Curve Lines BDC, BEC, is equal to that begat from CDB, therefore as 2; but that begat from the Fig. contained by the Curve Lines BDC, BFG, and the right Line CG, is as 3; therefore that begat from the Fig. contained by the Curve Lines BEC, BFG, and the right Line CG, is as 1; therefore $\frac{1}{8}$ of the Cylinder, and $\frac{1}{2}$ of that begat from CDA.

COROL. IV.

That begat from the aforeſaid Fig. to wit, that contained by the Curve Lines BEC, BFG, is equal to that begat from the Triline CGH; for ſince all the Applicates are equal, the Superficies in both Figures of the begat are equal; for Example, that begat from QS equal to that begat from RT, that begat from EF equal to that begat from FI, therefore the Solids begat are equal: Hence that begat from the Triline GHB is triple of that begat from the Triline CGH.

COROL. V.

That begat from BCH turned about CG is to the Cylinder begat as 7 to 8, for ſince that begat from the Triline CGH is to the Cylinder as 1 to 8, that begat from BCH will be to the Cylinder as 7 to 8.

COROL. VI.

Hence at laſt are had others begat, ſuppoſe from the Triline CFG; for ſince we have that begat from the Triline CGH, alſo that begat from the Triline, GFH, which is to that begat from the Rectangle under GH, IF, as the Triline to the Rectangle we have that begat from CFG, alſo from the Fig. contained by the Curves CDB, CFH, and by the right Line BH: For ſince that begat from the Semicircle CDB is to the Cylinder as 2 to 8, and that begat from BCH as 7 to 8; that begat from the aforeſaid Fig. to the Cylinder as 5 to 8, therefore equal to that begat from CBG, and ſince that begat from CEB is as 4, that begat from the Fig. contained by the Curves CEB, CFB, and the right Line BH will be to the Cylinder as 3 to 8, therefore equal to that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG. Hence thoſe begat from CDA, and from the Triline CFG, will be equal to that begat from the Triline BHF.

PROP. XXIX.

That begat from any right Segment of a Cycloid turned about the Baſe may be had; let ABF be turned about AF, that begat from DFB is equal to that begat from IYH, for they are homogene Figures, therefore the $\frac{1}{2}$ of the Cylinder of the ſame Baſe and Altitude by Prop. 4. But that begat from the Quadrant ADB is to the Cylinder of the ſame Baſe and Altitude as 2 to 3, therefore let AD be 7, DF 11; that begat from ABF will be to the Cylinder under the Altitude AF, as 10 $\frac{1}{6}$ to 18, or as the Cylinder under the ſame Baſe and Altitude compounded from $\frac{2}{3}$ of AD, and $\frac{1}{2}$ of DF, that is, as 61 to 108.

Let another Segment be KBN, ſince we have the Ratio of KBN to the Rectang. under KM, KB, and the Diſtance of the Center of Gravity of both from KM; we have the Ratio compounded from the Ratio's of the Diſtances, and of the Figures, but this is the Ratio of the Solids begat by Poſition 9; the ſame may be ſaid of any other Segment; for Example, of OBS.

COROL.

Hence are had thoſe begat from ADB, DEB, EFB apart; and that begat from EFB is equal to that begat from FIH; alſo that begat from BAF turned about BH, for we have the Diſtance of the Center of ABF from BH: Alſo of thoſe begat from DEB, EFB turned about BH; alſo from the Trapezium BAFH, and from any other from the general Rule delivered.

PROP. XXX.

We have the Center of Gravity of the begat as well from the Fig. of the Cycloid, as from any whole Segment turned about the Baſe, for it is in the Point in which the Baſe and the Axis are cut croſs-wiſe; to wit, the Center of the Solid begat is in the Axis, is drawn through the Centers of all the Planes; alſo in the Baſe of the Figures which is alſo drawn through the Centers of all the Circles; therefore the Center of the begat muſt needs be in the Concourſe or Meeting of both.

PROP. XXXI.

We have the Ratio of the Solid begat from the Fig. of the half Cycloid turned about the Axis to the Cylinder of the ſame Baſe and Altitude; let all be as above, the half Cycloid, the Fig. CBG turned about the Axis BC; that begat from it is to the Cylinder of the ſame Baſe and Altitude, as the Solid under the Altitude CB, proceeding by the

Squares CG, OS, AF, KN, *&c.* to the Parallelipiped under the Altitude CB, and of the Baſe from the Quadrate CG; for as the Parallelipiped is homogene to the Cylinder, ſo the aforeſaid Solid, proceeding by the Squares homogene to that begat from CBG: But the Ratio is known of the aforeſaid Solid to the Parallelipiped by Coroll. 20. of Prop. 22. therefore from thence is had the Ratio of the aforeſaid begat to the Cylinder, which we ſhall prove by Calculations beneath.

COROL. I.

Hence are had many Solids begat, being made by the Revolution about BC, to wit, that begat from the Triline BHG, alſo that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG, by taking away that begat from BEC the double of that begat from BDC; likewiſe that begat from CAFG; for if from the Parallelipiped under the Altitude CA, and of the Baſe from the Square of CG be taken away twice, the Iſoparallel under the Triline Baſe IGF, and the Altitude CG, leſs by the Solid under the Altitude IG, proceeding by the Squares under FI, ST, and from other Differences of the Arches and Sines, the Remainer will proceed by the Squares CG, OS, AF, and theſe are known by Coroll. 20. of Prop. 22. And it is as the aforeſaid Parallelipiped to the aforeſaid Remainer, ſo the Cylinder begat from CI, to another Solid, this it ſelf will be that begat from CAFG, which is homogene to the aforeſaid Remainer.

COROL.

COROL. II.

Hence is had the Center of Gravity of CBG, to wit, we have already the Diſtance of it from the Baſe CG, by Prop. 26. the Ratio of the Begetters is had which is compounded from the Ratio of the Figures which is had, and from the Ratio of Diſtances of which one is had, to wit, the half CG, therefore the other alſo is had, for from two compounded Terms being given, alſo from the two other of the compounding Ratio, ſince to the third Term of one is had, the fourth Term of the ſame, as is ſhewn above: Therefore is had the Perpendicular falling from the Center of the Fig. on the Baſe CG, alſo the Perpendicular falling from the ſame on the Axis CB; therefore is had the Concourſe of both, in which is the Center of the Fig. CBG.

COROL. III.

Alſo the Center of the Triline HBG is had, which may be demonſtrated in the ſame Manner, alſo that begat from the ſame Triline turned about GH; for we have the Ratio compounded of the Diſtances, and of the Figures, therefore of the Solids begat: Alſo we have the Center of the Trapezium CAFG, to wit, the Perpendicular falling from it on the Axis CB; therefore that begat from the ſame Trapezium turned about GI, which being taken from the Cylinder begat from GA, we have that begat from the Triline IFG.

PROP. XXXII.

The Solid begat from the Segment turned about the Axis is had, whoſe Baſe will fall in the Center of the generating Circle; (for Example) let ABF be the begat from it, it is Homogene to the Solid under the Altitude AB, proceeding by the Squares under the Compounds from the Arches and Sines, as appears; for AF is compounded from the Sine AD, and from the Arch DB; alſo KN from the Sine KL, and from the Arch LB, &c. And the Ratio of this Solid will be known to the Parallelipiped of the ſame Altitude and Baſe, therefore alſo of the aforeſaid begat from ABF to the Cylinder: I ſay, the aforeſaid Ratio will be known by Coroll. 7. of Prop. 14. and Coroll. 20. of Prop. 22.

COROL. I.

Hence is had the Center of Gravity of the Segment ABF, which may alſo be had in another Manner, to wit, from the Center being known of the whole Fig. CBG, and of the Trapezium CAFG; alſo that begat from BFC is had, *viz.* the double of that begat from ABF.

COROL. II.

That begat from DFB is had, to wit, by taking away the begat from ADB: And therefore the Center of the ſame DFB, alſo that begat from the Fig. contained by the Curves BDC, BFC, and the ſame Center of the Fig. Alſo that begat from the Triline EFB, by taking away that begat from or by AEB, the double of the begat from ADB, and the ſame Center of the Triline; alſo that begat from the Lunula contained by the Curves BEC, BFC by taking away the begat from or by BEC; alſo the Centers of the Figures are had, by the Begats being known.

COROL. III.

And from the Revolution made about HG, is had that begat from the Triline GHB, of which the Center is had; alſo from the Fig. CBG, by reaſon of the ſame Ratio; alſo that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG; alſo from the Fig. contained by the Curves BEC, BFG, and the right Line CG; alſo that begat from the aforeſaid Lunula; alſo from the Triline CGF or BHF, to wit, becauſe the Centers are known.

PROP. XXXIII. Fig. 16.

From a Solid being cut which proceeds by Rectangles under the right Sines and Complements, the Ratio of the Segments is had; let the aforeſaid Solid be ABDC proceeding by the Segments BAC, OHG, &c. or by Rectangles parallel to EG, which is under the right Sine and the Complement, as is ſhewn in Coroll. 2. of Prop. 13. let it be cut by the Plane OHG, ſince it is Homogene ASD, the Segments of the Solid are as the Segments of the Triang. And theſe are as the Squares of the Segments of the Baſe AB, BD, therefore by knowing the whole Solid, the Segments will be known.

COROL. I.

Hence from the Solid being cut by the Plane EG we may know the Segment intercepted by the Planes BAC, OHG, from which, if we take away the Iſoparallel under the Baſe EFA, and the Altitude AH, the Reſidue will be known, of which the Baſe is EG, and the Altitude EB.

COROL. II. Fig. 17.

Hence the Segments of the Solid AEC cut by the Plane DB will be known; to wit, the Iſoparallels had under the Baſe from the Trapezium ALKB, and the Altitude AG: But the Segment under the Baſe GIDH, and the Altitude GF may be had thus; the Solid will be known proceeding by the Sectors parallel to GID towards FE, it is Homogene to the Triangle, therefore the Half of the Iſoparallel under the Baſe from the Sector GID, and the Altitude GF; the Solid alſo will be known proceeding by Triangles parallel to GHD towards FE, for it is ſubduple of the Solid known proceeding by Rectangles under the right Sines, and of the Complements, by Coroll. 1. Therefore the Segment will be known, intercepted by the Planes AE, and BK, which being taken from the whole Solid AEC, we have the remaining Segment DBC.

COROL. III.

Hence the Solid Segment under the Baſe GHDI, and the Altitude GF, is equal to the Iſoparallel under the Baſe KLNP, and the Altitude LI.

COROL. IV. Fig. 24.

If we return to the Fig. CBG, the Solid proceeding or growing up by Rectangles under the Sines and the Arches, that is, under AD, DF, KL, LN, *&c.* is Homogene to the aforeſaid Solid: Hence if it be cut by the Plane LN, the Ratio of the Segments will be had as above, and by adding the Solid proceeding by the Squares of the Sines AD, KL being known, the Solid is had proceeding by the Rectangles under AF, AD, KN, KL, that is, under the Sines and the Compounds from the Sines and Arches; alſo the Ratio of the Segments is had by drawing the Plane KN.

PROP. XXXIV.

From the Revolution being made about the Axis is had that begat from the Fig. contained by $\frac{1}{2}$ the Cycloid, and the right Line the Subtenſe of the ſame: Let CIG be $\frac{1}{2}$ the Cycloid, let the Fig. contained be the Curve BFG, and the right Line BG, the Solid will be had from it begat by being turned about BC, to wit, if from the Solid begat from CBFG, we take away the Cone begat from the Triang. CBG, the Reſidue will give that begat from the aforeſaid Fig.

COROL. I.

Hence is had the Center of Gravity of the aforeſaid Fig. alſo of that begat from ♂ F B, to wit, by taking away the begat from A ♂ B being known, from that begat from AFB alſo known: Hence is had that begat from ♂ F G; alſo the Center of Gravity as well of ♂ F B as of ♂ F G.

COROL. II.

If there be another Line of $\frac{1}{2}$ the Cycloid GDB, the common Subtenſe BG, the Center of both together is in AF, ſince both the Applicates are parallel to AF, and equidiſtant, they are equal therefore in the Point ♂, as appears.

COROL. III.

Hence is had that begat from both taken together, for it is to the Cylinder begat from BG in the Ratio compounded of the Figures and of the Diſtances; the Diſtances are equal, for the Center of both Figures is in ♂, therefore thoſe begat are as the Figures: The Rectang. is double of the Fig. therefore that begat from the Fig. is ſubduple of that begat from the Rectang.

COROL. IV.

Hence that begat from the aforeſaid Fig. is *ſeſquialter* of the Cone begat from the Triang. BCG, and *ſubſeſquitertium* of that begat from the Triangle BGH: Hence that begat from both the Trilines taken together, is equal to that begat from the Fig. And the begats are as the Figures, ſince the Center of both is in ♂.

COROL. V.

If we turn the aforeſaid Fig. about CG, ſince that begat from CBG is to the Cylinder as 5 to 8, that is, as 15 to 24; that begat from the aforeſaid Fig. will be as 12 to 24, therefore that begat from the Triline BCG as 3: And that begat from the Triline GBH as 9. Hence that begat from $\frac{1}{2}$ the Fig. contained by the Curve BFG, and the right Line BG as 7; becauſe that begat from the Triang. GBH is as 16, therefore that begat from the other $\frac{1}{2}$ as 5: Hence you ſee the Proceſſion by odd Numbers, from the beginning being drawn from the Triline BCG, 3, 5, 7, 9: Laſtly, that begat from HGB as 21. Moreover, in this the two Trilines CBG, GBH agree with the Parabolicks, becauſe that begat from one is triple of that begat from the other.

SCHOLIUM. Fig. 13.

But I come to Calculations; Let the whole Cycloid be BAE, for altho' above it hath been inſtead of the Fig. of Sines, now I ſuppoſe it to be a Cycloid which is turned about BE: As CA begets a Circle, ſo alſo all the other ordinate Applicates parallels: Let CA, the Axis of the Fig. (for Example) be 14, EB the Baſe 44, according to the *Archimedean* Ratio; let the Homogene Solid ariſing or proceeding by the Squares under the Diameters of the Circles begat from the aforeſaid ordinate Applicates under the Altitude EB be called the greater Homogene; let there be another under the Altitude

CE, proceeding or ariſing by the Squares of the Applicates Parallels to CA, call it the leſſer Homogene: This is ſuboctuple of the greater, as appears; let the Iſoparallel be under the Baſe BAE, and the Altitude AI cut by the Plane EIB, the leſſer Homogene is equal to the upper Fruſtum; ſince it is double of the half Fruſtum, to wit, the Homogene will proceed by Squares, and the half Fruſtum by Triangles, both under the ſame Altitude CE, and every Square is double of its Triangle. Moreover, let the Center of Gravity of the Fig. BAE be in K, ſo that CK be to CA as 5 to 12, the aforeſaid Fruſtum will be to the Iſoparallel as 5 to 12; therefore alſo the leſſer Homogene as 5 to 12; therefore the greater Homogene as 40 to 12, that is, as 10 to 3. Therefore ſince the Fig. EAB is equal to three Circles under the Diameter CA, the Circle under the Diameter double of CA will be to the Fig. CAB, as 4 to 3; therefore the Cylinder under the Baſe from the aforeſaid Circle, and the Altitude AI to the Iſoparallel as 4 to 3, therefore the greater Homogene to this Cylinder as 10 to 4, or as 5 to 2. But this Cylinder is to that begat from the Rectang. under BE, CA turned about EB, as CA to EB, that is, as 14 to 44, therefore the greater Homogene will be to that begat from the aforeſaid Rectang. as 35 to 44. But the Parallelipiped under the Baſe from the Square of the double of CA, and the Altitude CB, is to the aforeſaid begat as 14 to 11, that is, as 56 to 44, and to the greater Homogene as 56 to 35, that is, as 8 to 5: Therefore the greater Homogene to that begat from the Fig. EAB as 35 to 27 $\frac{1}{2}$, that is as 14 to 11, therefore the greater Homogene to the Cylinder under the

Baſe from the Circle of the Diameter CA, and the Altitude AI, will be as 40 to 4, that is, as 10 to 1; therefore that begat from EAB to the Cylinder under the Baſe from the generating Circle, and the Altitude CA, is as 55 to 14.

But let IY or DF be 11, CB 14, the Parallelipiped under the Baſe from the Square IY, or DF, and the Altitude IH will be 847, that drawn into three is	2541
The Cube under the Side IH, will be 343, which, drawn into three, makes	1029
The Solid proceeding or ariſing by the Squares of the Arches IY, XZ, &c. under the Altitude IH, will be 392, which, drawn into three, makes	1176
The Solid Homogene to the Sphere, proceeding by the Squares of the Sines under the Altitude IH, will be 228 $\frac{2}{3}$, which, drawn into three, makes	686
The Solid proceeding by Rectangles under the Arches and Sines, and under the ſame Altitude IH, will be 297 $\frac{1}{2}$, which, drawn into three, makes	892 $\frac{1}{2}$
The Difference of both Solids	206 $\frac{1}{2}$
This Difference being added to the laſt Solid, the Aggregate will be	1099
This Aggregate being taken from the Solid, proceeding by the Squares of the Arches, the Remainer will be	77

And this is the Solid proceeding or ariſing by the Squares of the Differences between the Arches and the Sines.

If from the Parallelipiped under the Square CG, and the Altitude CA, we take away twice the Iſoparallel under the Baſe from the Triline GIF and the Altitude CG, leſs by the Solid proceeding by the Squares FI, ST, to wit, by the Squares of the Differences, as above, the Remainer will be the Solid, proceeding by the Squares CG, OS, AF, to wit 9779

Being added to the Solid proceeding by the Squares of the Arches; twice the Solid proceeding by the Rectangles under the Arches and Sines, leſs by the Solid proceeding by the Squares of the Sines, and the Aggregate proceeding by the Squares AF, KN, to wit, compounded from the Arches, and the Sines will be 2275

One being added to the other, and will be the Solid proceeding by the Squares CG, AF, *&c.* 12054

And the Parallelipiped under the Baſe from the Quadrate CG, and the Altitude CB, is 20328

Therefore the Parallelipiped is to the Solid proceeding by the Squares CG, AF, *&c.* that is the Cylinder begat from the Rectangle CH, to that begat from the Fig. CBG turned about BC, as 484 to 287.

Hence if we take away from that begat from the Fig. CBG, that begat from the Triangle CBG, that is $\frac{1}{3}$ of that begat from CH, the Remainer will give the begat from the Fig. contained by the Curve Line BFG, and the right Line BG, to wit $125\frac{2}{3}$; therefore

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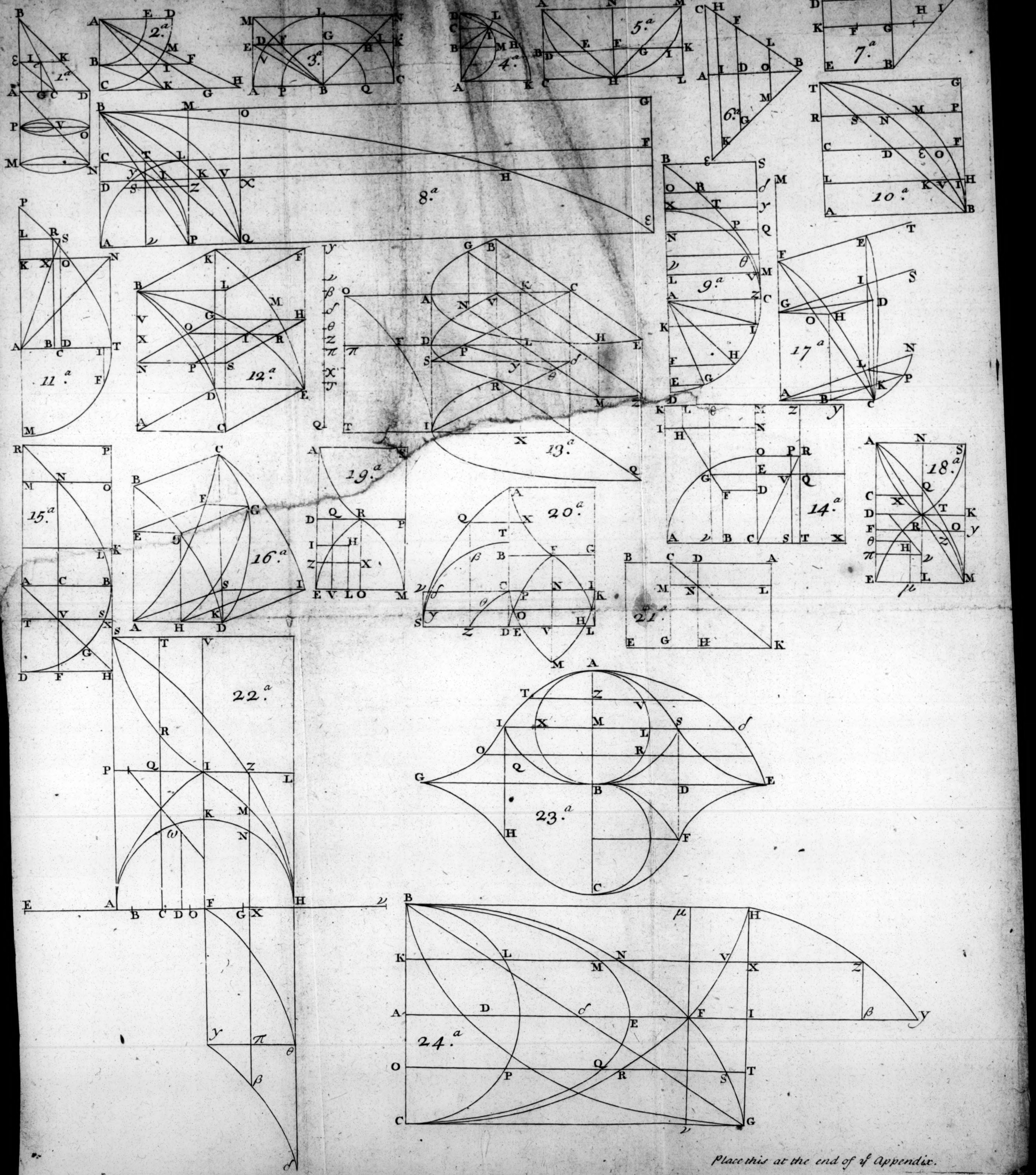

Place this at the end of ye Appendix.

therefore that begat from the other $\frac{1}{2}$ Portion 116 $\frac{1}{3}$: Therefore that begat from the Triline BCG 45, therefore that begat alſo from the other Triline 197.

And by taking away twice the Solid proceeding by the Squares of the Sines, from the Solid which proceeds by the Squares of CG, AF, the Remainer will be 10682; therefore if that begat from CDFG is 287, that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG will be 254 $\frac{1}{3}$; and conſequently that begat from the Semicircle CDB 32; therefore that begat from CEB 130 $\frac{2}{3}$; therefore that begat from the Fig. contained by the Curves BEC, BFG, and the right Line CG, is 56 $\frac{1}{3}$; and that begat from CH turned about CB to that begat about CG, as 847 to 539, that is 484 to 308; therefore that begat from CBG about BC, to that begat from CBG about CG, as 287 to 192 $\frac{1}{2}$: That begat from the Triline BCG is had in another Manner, to wit, by taking away that begat from the Fig. contained by the Curves BFG, BPG, half of that begat from CH, from that begat from CBG, that is 242 from 287, the Remainer 45 will be that begat from the Triline BCG.

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